Online Appendix for "Child skill production: Accounting for parental and market-based time and goods investments"

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A Analytical Issues

A.1 Separating the household's problem into an intratemporal and intertemporal problem

This appendix focuses on the case in which both parents work (i.e., $h_{m,t} > 0$ and $h_{f,t} > 0$). It also considers the family decision problem under uncertainty about children's future abilities or future parental wages and income. Importantly, this uncertainty has no effect on the intratermporal problem of subsection A.2. Under our main assumptions, uncertainty about children's ability also has no effect on the intertemporal problem of subsection A.3. In the absence of borrowing constraints, uncertainty about future parental wages and income would affect consumption and, therefore, total investment behavior due to precautionary savings motives; however, such uncertainty would not affect decisions during periods in which families are borrowing constrained. We briefly discuss these implications for our characterization of intertemporal decision making (under full certainty) in subsection A.3.

A.1.1 Full problem

The household's problem for periods $t = 1, ..., T$, is given by:

$$\begin{aligned} & V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) \\ & \quad = \max_{l_{m,t},\tau_{m,t},l_{f,t},\tau_{f,t},g_t,Y_{c,t},A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta \mathbb{E}_t V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \end{aligned}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$, $l_{j,t} \geq 0$ and $l_{j,t} + \tau_{j,t} \leq 1$ for $j = m, f$, child human capital production equation (1),

$$\begin{aligned} c_t + p_t g_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + P_{c,t}Y_{c,t} + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1-l_{m,t}) + W_{f,t}(1-l_{f,t}), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta_{T+1}, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}). \end{aligned}$$

We assume $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v_j'(\cdot) > 0$, and $v_j''(\cdot) \leq 0$, $j = m, f$. We also assume standard Inada conditions for preferences over consumption and leisure and that $\tilde{V}$ is strictly increasing and strictly concave in child skill and parental assets.[59] Expectations at time t, denoted by $\mathbb{E}_t$, implicitly integrate over future realizations of children's ability, parental wages, and family income conditional on the current state.

Suppose both parents work in the market, $l_{j,t} + \tau_{j,t} < 1$, $j = m, f$. Let λ_t be the Lagrange multiplier on the period t budget constraint and ξ_t be the Lagrange multiplier on the period t borrowing constraint.

[59] The continuation value, $\tilde{V}$, also depends on all future non-labor income, which we suppress here for ease of notation.

The first order conditions for c_t, $\tau_{j,t}$, g_t, $Y_{c,t}$, $l_{j,t}$, A_{t+1}, $j=m,f$ are:

$$\begin{aligned}
\lambda_t &= u'(c_t), & (20)\\
\lambda_t W_{j,t} &= \beta\frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}}\frac{\partial \mathcal{H}_t}{\partial f_t}\frac{\partial f_t}{\partial \tau_{j,t}}, \quad j=m,f, & (21)\\
\lambda_t p_t &= \beta\frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}}\frac{\partial \mathcal{H}_t}{\partial f_t}\frac{\partial f_t}{\partial g_t}, & (22)\\
\lambda_t P_{c,t} &= \beta\frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}}\frac{\partial \mathcal{H}_t}{\partial f_t}\frac{\partial f_t}{\partial Y_{c,t}}, & (23)\\
v_j'(l_{j,t}) &= \lambda_t W_{j,t}, \quad j=m,f, & (24)\\
\lambda_t+\xi_t &= \mathbb{E}_t\left[\lambda_{t+1}\beta(1+r)\right]. & (25)
\end{aligned}$$

Combining the first order conditions for consumption and leisure yields the standard result that the marginal rate of substitution equals the wage rate:

$$v'(l_{j,t}) = u'(c_t)W_{j,t},\ j=m,f. \tag{26}$$

We also have:

$$\begin{aligned}
\lambda_t(c_t+p_tg_t+P_{c,t}Y_{c,t}+A_{t+1}-(1+r)A_t-y_t-W_{m,t}(1-l_{m,t}-\tau_{m,t})-W_{f,t}(1-l_{f,t}-\tau_{f,t})) &= 0, & (27)\\
\xi_t(A_{t+1}-A_{min,t}) &= 0. & (28)
\end{aligned}$$

Note that if a parent does not work, the cost of child time investment is measured by the value of lost leisure, and $v_j'(l_{j,t}) = \beta\frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}}\frac{\partial \mathcal{H}_t}{\partial f_t}\frac{\partial f_t}{\partial \tau_{j,t}},\ j=m,f$.

A.1.2 Intratemporal problem

For $h_{m,t}>0$ and $h_{f,t}>0$, the intratemporal problem minimizes expenditures, given X_t:

$$\min_{g_t,\tau_{m,t},\tau_{f,t},Y_{c,t}} p_tg_t+P_{c,t}Y_{c,t}+W_{m,t}\tau_{m,t}+W_{f,t}\tau_{f,t}$$

subject to non-negative inputs $(\tau_{m,t},\tau_{f,t},g_t,Y_{c,t})$, $\tau_{m,t}<1$, $\tau_{f,t}<1$, and $X_t=f_t(\tau_{m,t},\tau_{f,t},g_t,Y_{c,t};H_m,H_f)$. Let $\bar{p}_t$ be the Lagrange multiplier on this constraint. The first order conditions for $\tau_{j,t}$, g_t, and $Y_{c,t}$, $j=m,f$ are:

$$\begin{aligned}
W_{j,t} &= \bar{p}_t\frac{\partial f_t}{\partial \tau_{j,t}}, \quad j=m,f, & (29)\\
p_t &= \bar{p}_t\frac{\partial f_t}{\partial g_t}, & (30)\\
P_{c,t} &= \bar{p}_t\frac{\partial f_t}{\partial Y_{c,t}}. & (31)
\end{aligned}$$

Substitute these first order conditions into the minimand:

$$E_t=\bar{p}_t\left[g_t\frac{\partial f_t}{\partial g_t}+Y_{c,t}\frac{\partial f_t}{\partial Y_{c,t}}+\tau_{m,t}\frac{\partial f_t}{\partial \tau_{m,t}}++\tau_{f,t}\frac{\partial f_t}{\partial \tau_{f,t}}\right].$$

Because $f_t(\tau_{m,t},\tau_{f,t},g_t,Y_{c,t})$ is homogenous of degree 1 (Constant Returns to Scale), we have:

$$X_t=f_t(\tau_{m,t},\tau_{f,t},g_t,Y_{c,t})=\frac{\partial f_t}{\partial g_t}g_t+\frac{\partial f_t}{\partial \tau_{m,t}}\tau_{m,t}+\frac{\partial f_t}{\partial \tau_{f,t}}\tau_{f,t}+\frac{\partial f_t}{\partial Y_{c,t}}Y_{c,t},$$

and, $E_t=\bar{p}_tX_t$.

A.1.3 Intertemporal problem

Suppose in every period, $t = 1, ..., T$, along with leisure and assets, the household chooses an amount of child investment X_t, given a per period composite price $\bar{p}_t$. This problem can be written as follows:

$$V_t(\theta_t, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) = \max_{l_{m,t}, l_{f,t}, X_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta \mathbb{E}_t V_{t+1}(\theta_{t+1}, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})$$

subject to $0 \leq l_{m,t}, l_{f,t} \leq 1$, $X_t \geq 0$,

$$\begin{aligned}
c_t + \bar{p}_t(\Pi_t, H_m, H_f)X_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}), \\
\Psi_{t+1} &= \mathcal{H}_t\left(X_t, \theta_t, \Psi_t\right), \\
A_{t+1} &\geq A_{min,t}, \\
V_{T+1}(\theta_{T+1}, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}).
\end{aligned}$$

The first order conditions for c_t, $l_{j,t}$, X_t, A_{t+1}, $j = m, f$ are:

$$\lambda_t = u'(c_t), \tag{32}$$

$$v_j'(l_{j,t}) = \lambda_t W_{j,t}, \quad j = m, f, \tag{33}$$

$$\lambda_t \bar{p}_t = \beta \frac{\partial \mathbb{E}_t V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \mathcal{H}_t}{\partial X_t}, \tag{34}$$

$$\lambda_t + \xi_t = \mathbb{E}_t\left[\lambda_{t+1}\beta(1+r)\right]. \tag{35}$$

We also have:

$$\lambda_t(c_t + \bar{p}_t(\Pi_t, H_m, H_f)X_t + A_{t+1} - (1+r)A_t - y_t - W_{m,t}(1 - l_{m,t}) - W_{f,t}(1 - l_{f,t})) = 0, \tag{36}$$

$$\xi_t(A_{t+1} - A_{min,t}) = 0. \tag{37}$$

Comparing first order conditions, we see the separated problem has first order Conditions (32), (33), (35), and (37) corresponding to the full problem Conditions (20), (24), (25), and (28). If we substitute $\bar{p}_t X_t = p_t g_t + P_{c,t} Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$, into Condition (36), we have Condition (52). Lastly, noting that $X_t = f_t$, substituting $\bar{p}_t$ from Conditions (29), (30), and (31), separately into Condition (34), yields the full problem Conditions (21), (22), and (23).

A.2 Characterizing the Intratemporal Problem

Given the static nature of the intratemporal problem, we drop time t subscripts throughout this subsection. Because none of the results in this subsection depend on future values of child abilities, parental wages, or family income, uncertainty about their values also plays no role.

A.2.1 Parental skill neutrality

Notice that if $f(\tau_m, \tau_f, g, Y_c; H_m) = f(\tau_m H_m, \tau_f H_f, g, Y_c)$, then we can re-write equations (4) and (5) as follows:

$$\begin{aligned}
\tilde{w}_m \equiv \frac{w_m}{p} &= \frac{f_1(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)} \\
\tilde{w}_f \equiv \frac{w_f}{p} &= \frac{f_2(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)} \\
\tilde{P}_c \equiv \frac{P_c}{p} &= \frac{f_4(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)}{f_3(\Phi_m H_m, \Phi_f H_f, 1, \Phi_c)},
\end{aligned}$$

where $f_j(\cdot)$ reflects the partial derivative with respect to argument j. From these 3 equations, we can solve for "effective" input ratios $\Phi_m H_m$, $\Phi_f H_f$, and Φ_c as functions of relative prices $(\tilde{w}_m, \tilde{w}_f, \tilde{P}_c)$ and the technology $f(\cdot)$. Clearly, then, relative expenditure ratios $\tilde{w}_m H_m \Phi_m$, $\tilde{w}_f H_f \Phi_f$, and $\tilde{P}_c \Phi_c$ depend only on relative prices – and not parental human capital levels – as well. Because none of the relative expenditure ratios depend on parental human capital levels, expenditure shares must also be constant in parental human capital.

A.2.2 Some results for w_m and H_m with CES

Normalizing $\bar{a}_m = \bar{a}_g = a_{Y_c} = 1$, we have the following for single mothers:

$$\begin{aligned} f(\tau_m, g, Y_c; H_m) &= \left[([\varphi_m(H_m)\tau_m]^\rho + [\varphi(H_m)g]^\rho)^{\gamma/\rho} + Y_c^\gamma \right]^{1/\gamma} \\ \Phi_m &= \left[\frac{\varphi_g(H_m)}{\varphi_m(H_m)} \right]^{\rho/(\rho-1)} \tilde{W}_m^{1/(\rho-1)} \\ \Phi_c &= \varphi_g(H_m)^{\frac{\rho}{\gamma-1}} \left[(\varphi_m(H_m)\Phi_m)^\rho + \varphi_g(H_m)^\rho \right]^{\frac{\gamma-\rho}{\rho(\gamma-1)}} \tilde{P}_c^{\frac{1}{\gamma-1}}. \end{aligned}$$

If we define elasticities $\bar{\varphi}_j = \varphi_j'(H_m) H_m / \varphi_j(H_m)$ for $j = m, g$, then

$$\begin{aligned} \frac{\partial \Phi_m}{\partial w_m} &= -\left(\frac{1}{1-\rho} \right) \Phi_m w_m^{-1} && (38) \\ \frac{\partial \Phi_m}{\partial H_m} &= -\left(\frac{1}{1-\rho} \right) \Phi_m H_m^{-1} [1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m)] && (39) \\ &= \frac{\partial \Phi_m}{\partial w_m} \frac{w_m}{H_m} [1 + \rho(\bar{\varphi}_g - \bar{\varphi}_m)] && (40) \end{aligned}$$

Thus, the ratio of mother's time to goods inputs, Φ_m, does not depend on H_m if $\bar{\varphi}_g = 0$ and $\bar{\varphi}_m = 1/\rho$.

Next, consider the ratio of child care to goods inputs, letting $\chi(H_m) \equiv [\varphi_m(H_m)\Phi_m(H_m)]^\rho + \varphi_g(H_m)^\rho$:

$$\begin{aligned} \frac{\partial \Phi_c}{\partial H_m} &= \left(\frac{\rho}{\gamma-1} \right) \Phi_c \bar{\varphi}_g H_m^{-1} + \left(\frac{\gamma-\rho}{\rho(\gamma-1)} \right) \Phi_c \frac{\chi'(H_m)}{\chi} \\ &= \left(\frac{\rho}{\gamma-1} \right) \Phi_c \bar{\varphi}_g H_m^{-1} + \left(\frac{\gamma-\rho}{(1-\gamma)(1-\rho)} \right) \Phi_c H_m^{-1} \left[1 + \rho\bar{\varphi}_g - \bar{\varphi}_m - \frac{\varphi_g^\rho (1 - \bar{\varphi}_m - \bar{\varphi}_g)}{(\varphi_m \Phi_m)^\rho + \varphi_g^\rho} \right], && (41) \end{aligned}$$

where the second equality uses equation (39). When parental skills do not affect the productivity of goods inputs (i.e., $\bar{\varphi}_g = 0$), this simplifies considerably to

$$\frac{\partial \Phi_c}{\partial H_m} = \left(\frac{\gamma-\rho}{(1-\gamma)(1-\rho)} \right) \left[\frac{(\varphi_m \Phi_m)^\rho}{(\varphi_m \Phi_m)^\rho + \varphi_g^\rho} \right] \Phi_c H_m^{-1} (1 - \bar{\varphi}_m)$$

In this case, the ratio of child care services to goods inputs, Φ_c, does not depend on H_m if $\gamma = \rho$ or $\bar{\varphi}_m = 1$.

Now, consider the effects of H_m on the ratios of expenditures (or expenditure shares):

$$\begin{aligned} \frac{\partial(\tilde{W}_m \Phi_m)}{\partial H_m} &= w_m \Phi_m + \tilde{W}_m \frac{\partial \Phi_m}{\partial H_m} \\ &= -w_m \Phi_m \left(\frac{\rho}{1-\rho} \right) (1 + \bar{\varphi}_g - \bar{\varphi}_m) \\ \frac{\partial(\tilde{P}_c \Phi_c)}{\partial H_m} &= \tilde{P}_c \frac{\partial \Phi_c}{\partial H_m}, \end{aligned}$$

where $\frac{\partial \Phi_c}{\partial H_m}$ is given by equation (41). If $\bar{\varphi}_g = 0$ and $\bar{\varphi}_m = 1$, then the ratio of expenditures for any pair of inputs does not depend on H_m, in which case expenditure shares are also independent of maternal human capital. Regardless of $\bar{\varphi}_m$ and $\bar{\varphi}_g$, the ratio of expenditures on maternal time relative to goods inputs does not depend on H_m if $\rho = 0$, while the ratio of expenditures on child care relative to home goods inputs does not depend on H_m if $\rho = \gamma = 0$. Thus, if $\rho = \gamma = 0$ (i.e., $f_t(\cdot)$ is Cobb-Douglas in all inputs), all expenditure shares are independent of H_m.

A.2.3 Comparative statics results for expenditure shares

For simplicity, we consider the case of single mothers and drop all time subscripts (as we focus on within-period relationships), so

$$f = \left[(a_m \tau_m^{\rho} + a_g g^{\rho})^{\gamma/\rho} + a_{Yc} Y_c^{\gamma} \right]^{1/\gamma}. \tag{42}$$

Total investment expenditures are $E = pg + P_c Y_c + W_m \tau_m = g\,(p + P_c \Phi_c + W_m \Phi_m)$, where the latter follows from Equations (8) and (9). Expenditure shares are given by:

$$S_g \equiv \frac{pg}{E} = \frac{p}{p + P_c\Phi_c + W_m\Phi_m}, \quad S_{\tau_m} \equiv \frac{W_m\tau_m}{E} = \frac{W_m\Phi_m}{p + P_c\Phi_c + W_m\Phi_m}, \quad S_{Yc} \equiv \frac{P_c Y_c}{E} = \frac{P_c\Phi_c}{p + P_c\Phi_c + W_m\Phi_m},$$

where Φ_m and Φ_c are implicitly defined by Equations (8) and (9). Throughout this subsection of the Appendix, define $D \equiv p + P_c\Phi_c + w_m H_m \Phi_m$.

The following proposition characterizes the effects of child care prices on expenditure shares.

Proposition 4. *If and only if $\gamma < 0$, then P_c has strictly positive own-price effects on S_{Y_c} and strictly negative cross-price effects on S_g and S_{τ_m}.*

Proof of Proposition 4: We can differentiate shares with respect to P_c:

$$\frac{\partial S_g}{\partial P_c} = \frac{\gamma p \Phi_c}{(1-\gamma)D^2}, \qquad \frac{\partial S_\tau}{\partial P_c} = \frac{\gamma w_m H_m \Phi_m \Phi_c}{(1-\gamma)D^2}, \qquad \frac{\partial S_{Yc}}{\partial P_c} = \frac{-\gamma[pg + w_m H_m \tau]\Phi_c}{(1-\gamma)D^2}.$$

The stated results in Proposition 4 are immediate from these derivatives. □

Given the nested nature of $f(\cdot)$, the impacts of price changes on home inputs g and τ_m are slightly more complicated, though symmetric.

Proposition 5. *Expenditure shares on home inputs (g or τ_m) are strictly decreasing in their own price (p or w_m) if $\min\{\rho, \gamma\} > 0$ and strictly increasing in their own price if $\max\{\rho, \gamma\} < 0$. Expenditure shares on home inputs are strictly decreasing in the other home input price if $\rho < \min\{0, \gamma\}$, and strictly increasing in the other home input price if $\rho > \max\{0, \gamma\}$. The expenditure share on market child care services is strictly increasing in the price of both home inputs if and only if $\gamma > 0$.*

Proof of Proposition 5: We can differentiate expenditure shares with respect to p:

$$\begin{aligned}
\frac{\partial S_g}{\partial p} &= \frac{-\left\{\rho(1-\gamma)[P_c\Phi_c a_m \Phi_m^{\rho} + w_m H_m \Phi_m (a_m \Phi_m^{\rho} + a_g)] + \gamma(1-\rho)P_c\Phi_c a_g\right\}}{(1-\gamma)(1-\rho)[a_m\Phi_m^{\rho} + a_g]D^2} \\
\frac{\partial S_\tau}{\partial p} &= \frac{w_m H_m \Phi_m \left\{p\rho(1-\gamma)[a_m\Phi_m^{\rho} + a_g] + P_c\Phi_c(\rho-\gamma)a_g\right\}}{p(1-\rho)(1-\gamma)[a_m\Phi_m^{\rho} + a_g]D^2} \\
\frac{\partial S_{Yc}}{\partial p} &= \frac{\gamma P_c \Phi_c a_g \left\{p + w_m H_m \Phi_m\right\}}{p(1-\gamma)[a_m\Phi_m^{\rho} + a_g]D^2},
\end{aligned}$$

and with respect to w_m:

$$\begin{aligned}
\frac{\partial S_g}{\partial w_m} &= \frac{p\left\{P_c\Phi_c(\rho-\gamma)a_m\Phi_m^\rho+\rho w_m H_m\Phi_m(1-\gamma)[a_m\Phi_m^\rho+a_g]\right\}}{D^2 w_m(1-\gamma)(1-\rho)[a_m\Phi_m^\rho+a_g]}\\
\frac{\partial S_\tau}{\partial w_m} &= -\frac{H_m\Phi_m\left\{p\rho(1-\gamma)[a_m\Phi_m^\rho+a_g]+P_c\Phi_c[\gamma(1-\rho)a_m\Phi_m^\rho+\rho(1-\gamma)a_g]\right\}}{(1-\rho)(1-\gamma)[a_m\Phi_m^\rho+a_g]D^2}\\
\frac{\partial S_{Yc}}{\partial w_m} &= \frac{\gamma P_c p\Phi_c a_m\Phi_m^\rho}{a_g w_m(1-\gamma)D^2}.
\end{aligned}$$

The stated results in Proposition 5 are immediate from these derivatives. □

Complementarity between both home inputs ($\rho < 0$) and between the home composite input and market child care ($\gamma < 0$) ensures that substitution out of a home input whose price rises is insufficient to compensate for the higher price, leading to a greater expenditure share on that input. If home inputs are not only complementary ($\rho < 0$) but also more complementary than home inputs with market child care ($\rho < \gamma$), then an increase in the price of one home input will cause the expenditure share of the other to fall. The converse of these statements applies when inputs are substitutes. Finally, substitutability between home and market inputs ($\gamma > 0$) implies that an increase in either home input will raise the share of expenditures on child care, while complementarity ($\gamma < 0$) implies the opposite.

In considering the role of parental skills, we assume the following convenient functional forms:

$$\varphi_m(H_m)=H_m^{\bar\varphi_m} \qquad \text{and} \qquad \varphi_g(H_m)=H_m^{\bar\varphi_g}, \tag{43}$$

where the exponents $\bar\varphi_m \geq 0$ and $\bar\varphi_g \geq 0$ determine the returns to scale of parental human capital in the production of child skills. Note that the $\bar\varphi_j$ here correspond to the elasticities in Section A.2.2. The overall implications of $\bar\varphi_g > 0$ on expenditure shares is most transparent when the effect of maternal skills on the productivity of time investment is neutralized by assuming $\bar\varphi_m = 1$. The following proposition formally characterizes this case.

Proposition 6. *Suppose $\bar\varphi_m = 1$ and $\bar\varphi_g > 0$. (A) S_τ is strictly decreasing in H_m if $\rho > \max\{0,\gamma\}$, while it is strictly increasing in H_m if $\rho < \min\{0,\gamma\}$. (B) S_g is strictly decreasing in H_m if $\max\{\rho,\gamma\} < 0$, while it is strictly increasing in H_m if $\min\{\rho,\gamma\} > 0$. (C) S_{Y_c} is strictly decreasing in H_m if and only if $\gamma > 0$.*

Proof of Proposition 6: Differentiating D with respect to H_m yields:

$$\frac{\partial D}{\partial H_m} = \frac{P_c\Phi_c[a_m\Phi_m^\rho((\gamma-\rho)(1-\bar\varphi_m)+\rho(\gamma-1)\bar\varphi_g)+a_g(\rho-1)\gamma\bar\varphi_g]+w_m H_m\Phi_m\rho(\gamma-1)(1-\bar\varphi_m+\bar\varphi_g)[a_m\Phi_m^\rho+a_g]}{(1-\gamma)(1-\rho)H_m[a_m\Phi_m^\rho+a_g]}$$

Using this, we have

$$\begin{aligned}
\frac{\partial S_g}{\partial H_m} &= \frac{-p\frac{\partial D}{\partial H_m}}{D^2}\\
\frac{\partial S_\tau}{\partial H_m} &= \frac{w_m\Phi_m p\rho(\gamma-1)(1-\bar\varphi_m+\bar\varphi_g)[a_m\Phi_m^\rho+a_g]}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho+a_g]D^2}\\
&\quad+\frac{w_m\Phi_m P_c\Phi_c\left(\gamma(\rho-1)(1-\bar\varphi_m)a_m\Phi_m^\rho+(\bar\varphi_g(\gamma-\rho)+\rho(\gamma-1)(1-\bar\varphi_m))a_g\right)}{(1-\gamma)(1-\rho)[a_m\Phi_m^\rho+a_g]D^2}\\
\frac{\partial S_{Yc}}{\partial H_m} &= \frac{\gamma P_c\Phi_c[pa_m\Phi_m^\rho(1-\bar\varphi_m-\bar\varphi_g)-pa_g\bar\varphi_g+w_m H_m\Phi_m a_m\Phi_m^\rho(1-\bar\varphi_m)]}{(1-\gamma)H_m[a_m\Phi_m^\rho+a_g]D^2}.
\end{aligned}$$

The stated results in Proposition 6 are immediate from these derivatives. □

A.3 Characterizing the Intertemporal Problem

A.3.1 Roles of Assumption 1 and 2

The first order condition for X_t is:

$$\beta \mathbb{E}_t \left[\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} \frac{\partial \Psi_{t+1}}{\partial X_t} \right] = \bar{p}_t u'(c_t). \tag{44}$$

Envelope conditions are

$$\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} = \beta \mathbb{E}_{t+1} \left[\frac{\partial V_{t+2}}{\partial \Psi_{t+2}} \frac{\partial \Psi_{t+2}}{\partial \Psi_{t+1}} \right] \quad t = 0, ..., T-1.$$

and

$$\frac{\partial V_{T+1}}{\partial \Psi_{T+1}} = \frac{\partial \tilde{V}}{\partial \Psi_{T+1}}.$$

Combining the envelope conditions for periods $t+1, \ldots, T+1$ and applying the law of iterated expectations gives

$$\frac{\partial V_{t+1}}{\partial \Psi_{t+1}} = \beta^{T-t} \mathbb{E}_{t+1} \left[\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} \right].$$

By substituting this into Equation (44), we get

$$\beta^{T-t+1} \mathbb{E}_t \left[\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \left(\prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} \right) \frac{\partial \Psi_{t+1}}{\partial X_t} \right] = \bar{p}_t u'(c_t). \tag{45}$$

Assumptions 1 and 2 considerably simply the expression in the expectation operator. Assumption 2 implies

$$\begin{aligned} \frac{\partial \Psi_{t+1}}{\partial X_t} &= \delta_1 \frac{\Psi_{t+1}}{X_t}, \\ \frac{\partial \Psi_{t+1}}{\partial \Psi_t} &= \delta_2 \frac{\Psi_{t+1}}{\Psi_t}, \end{aligned} \tag{46}$$

where the last condition leads to

$$\prod_{s=t+1}^{T} \frac{\partial \Psi_{s+1}}{\partial \Psi_s} = \delta_2^{T-t} \frac{\Psi_{T+1}}{\Psi_{t+1}}. \tag{47}$$

Substituting Equations (46) and (47) into Equation (45) yields

$$\frac{\beta^{T-t+1} \delta_2^{T-t} \delta_1}{X_t} \mathbb{E}_t \left[\Psi_{T+1} \frac{\partial \tilde{V}}{\partial \Psi_{T+1}} \right] = \bar{p}_t u'(c_t).$$

Under Assumption 1, $\frac{\partial \tilde{V}}{\partial \Psi_{T+1}} = \frac{\alpha}{\Psi_{T+1}}$, which implies Equation (10) when substituted into the above expression.[60]

This result makes clear that child ability levels, θ_t, do not impact investment – or any other – decisions due to log separability of θ_t from other inputs in the child production function and log preferences for child skills. As such, uncertainty about children's abilities has no affect on family decisions, or any results that follow.

[60] If $\tilde{V}$ is not logarithmic over final human capital, then the FOC for total investment each period depends on the final level of child skill, which in turn depends on *all* periods of investments, including the current period. This implies that each X_t FOC would generally be a nonlinear function of total investments from all periods, yielding a complex system of nonlinear equations to solve.

A.3.2 Total Expenditures

Uncertainty about future wages or income (but not child ability) would affect unconstrained intertemporal consumption allocations due to precautionary savings motives. Because incorporating this effect would greatly complicate the analysis for unconstrained families with little added insight and because this uncertainty would not impact the behavior of borrowing-constrained families, we abstract from uncertainty throughout the rest of subsection A.3.[61]

To characterize investment behavior when constraints are non-binding throughout parents' lives, we make a simplifying assumption on the continuation value function $\tilde{U}$. This assumption is not necessary for any results for borrowing constrained households.

Assumption 3. *$\tilde{U}(H_m, H_f, A) = \hat{U}((1+r)A + \chi_m H_m + \chi_f H_f)$ where the constants χ_m and χ_f are non-negative and $\hat{U}(\cdot)$ is strictly increasing and strictly concave.*

This assumption represents the case where parents at date $T+1$ value their remaining lifetime wealth as defined by current assets plus the discounted present value of all future earnings represented by $\chi_j H_j$.[62] For ease of exposition, we have suppressed dependence of $\tilde{U}$ and $\hat{U}$ on non-labor earnings, y_t. For constrained families, future income is irrelevant for current decisions. For unconstrained families, we will assume potential non-labor earnings until retirement at T_R. It is useful to define $\Delta(x) \equiv \hat{U}'(x)$, which is a strictly decreasing function given strict concavity of $\hat{U}(\cdot)$.

Lemma 1. *Consumption, c_t, is strictly increasing in parental human capital (H_m, H_f), current skill prices $(w_{m,t}, w_{f,t})$, and current non-labor income (y_t) with $\frac{\partial c_t}{\partial H_j} = \frac{\partial c_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$. Consumption, c_t, is independent of current and all future household goods and child care input prices, $\{p_\tau, P_{c,\tau}\}_{\tau=t}^{T}$. If borrowing constraints are non-binding in all periods $t, ..., T$, then consumption, c_t, is strictly increasing in all future skill prices and non-labor income, $\{w_{m,\tau}, w_{f,\tau}, y_\tau\}_{\tau=t}^{T}$.*

Proof of Lemma 1: As noted in the text, the budget constraint for constrained households is

$$c_t = (1+r)A_t + W_{m,t}(1 - L_{m,t}(u'(c_t)W_{m,t})) + W_{f,t}(1 - L_{f,t}(u'(c_t)W_{f,t})) + y_t - \frac{K_t}{u'(c_t)} - A_{min,t},$$

where we have defined $l_{j,t} = L_{j,t}(u'(c_t)W_{j,t}))$ for $j = m, f$. Applying the implicit function theorem yields the following: $\partial c_t / \partial p_t = \partial c_t / \partial P_{c,t} = 0$,

$$\begin{aligned}
\frac{\partial c_t}{\partial w_{j,t}} &= \frac{\left(1 - l_{j,t} - u'(c_t)W_{j,t}L'_{j,t}\right) H_j}{1 + u''(c_t)\left[W_{m,t}^2 L'_{m,t} + W_{f,t}^2 L'_{f,t}\right] - K_t \frac{u''(c_t)}{u'(c_t)^2}} > 0, \quad j \in \{m, f\}, \\
\frac{\partial c_t}{\partial y_t} &= \frac{1}{1 + u''(c_t)\left[W_{m,t}^2 L'_{m,t} + W_{f,t}^2 L'_{f,t}\right] - K_t \frac{u''(c_t)}{u'(c_t)^2}} > 0,
\end{aligned}$$

and $\frac{\partial c_t}{\partial H_j} = \frac{\partial c_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$.[63]

[61] Uncertainty about future wages and income has no effect on X_t and, therefore, specific investment inputs for borrowing-constrained families, because uncertainty only affects total investment X_t through consumption c_t, which is fully determined by current assets, prices, wages, and income for constrained families.

[62] For example, $\chi_j = \sum_{k=0}^{T_R-(T+1)} (1+r)^{-k} w_{T+1+k}$, assuming individuals retire at date T_R.

[63] The numerator of $\partial c_t / \partial w_{j,t}$ is positive, because $1 - l_{j,t} > 0$ and $L'_{j,t} < 0$, i.e. leisure falls when its marginal cost, $u'(c_t)W_{j,t}$ rises.

For unconstrained households, the convenient assumption that $\beta(1+r) = 1$ implies that $c_t = c$, for all t. This simplifies the expressions that follow without altering any important conclusions. Along with Assumption 3, $\beta(1+r) = 1$ implies that $A_{T+1} = \frac{\Delta^{-1}(u'(c_T)) - \chi_m H_m - \chi_f H_f}{1+r}$. As with the binding constraint case, we can now substitute these expressions into the lifecycle budget constraint and collect consumption terms to obtain:

$$\Upsilon_{T-t} c + (1+r)^{-(T+1-t)} \Delta^{-1}\Big(u'(c)\Big) + \frac{\bar{K}_t}{u'(c)} - \sum_{j=0}^{T-t} (1+r)^{-j} \Big[W_{m,t+j}(1 - L_{m,t+j}) + W_{f,t+j}(1 - L_{f,t+j}) \Big]$$

$$= (1+r)A_t + (1+r)^{-(T+1-t)} \Big[\chi_m H_m + \chi_f H_f \Big] + \sum_{j=0}^{T_R - t} (1+r)^{-j} y_{t+j},$$

where the constants $\Upsilon_{T-t} \equiv \sum_{j=0}^{T-t} (1+r)^{-j} > 0$ and $\bar{K}_t \equiv \sum_{j=0}^{T-t} (1+r)^{-j} K_{t+j} > 0$, and we recognize the dependence of leisure on its marginal cost, $L_{j,t}(u'(c)W_{j,t})$. This implicitly defines consumption as a function of current and future wages, non-labor income, parental human capital, period t assets, and other preference/technology parameters. We can then use the implicit function theorem to determine how prices, non-labor income, and parental human capital affect consumption. Letting π generically reflect these parameters,

$$\frac{\partial c}{\partial \pi} = \frac{\sum_{j=0}^{T-t} (1+r)^{-j} \left[\frac{\partial W_{m,t+j}}{\partial \pi} \Big(1 - l_{m,t+j} - u'(c) W_{m,t+j} L'_{m,t+j}\Big) + \frac{\partial W_{f,t+j}}{\partial \pi} \Big(1 - l_{f,t+j} - u'(c) W_{f,t+j} L'_{f,t+j}\Big) \right]}{\Upsilon_{T-t} + \sum_{j=0}^{T-t} (1+r)^{-j} u''(c) \Big[W^2_{m,t+j} L'_{m,t+j} + W^2_{f,t+j} L'_{f,t+j} \Big] - \bar{K}_t \frac{u''(c)}{u'(c)^2} + (1+r)^{-(T+1-t)} \frac{u''(c)}{\Delta'(\Delta^{-1}(u'(c)))}}$$

$$+ \frac{(1+r)^{-(T+1-t)} \left[\frac{\partial \chi_m}{\partial \pi} H_m + \chi_m \frac{\partial H_m}{\partial \pi} + \frac{\partial \chi_f}{\partial \pi} H_f + \chi_f \frac{\partial H_f}{\partial \pi} \right] + \sum_{j=0}^{T_R - t} (1+r)^{-j} \frac{\partial y_{t+j}}{\partial \pi}}{\Upsilon_{T-t} + \sum_{j=0}^{T-t} (1+r)^{-j} u''(c) \Big[W^2_{m,t+j} L'_{m,t+j} + W^2_{f,t+j} L'_{f,t+j} \Big] - \bar{K}_t \frac{u''(c)}{u'(c)^2} + (1+r)^{-(T+1-t)} \frac{u''(c)}{\Delta'(\Delta^{-1}(u'(c)))}}.$$

The denominator is strictly positive, because $L'_{k,t} < 0$, $u''(\cdot) < 0$, $\bar{K}_t > 0$, and $\Delta'(\cdot) < 0$. Furthermore, $l_{k,t} < 1$ and $L'_{k,t} < 0$ implies that the numerator terms $\Big(1 - l_{k,t+j} - u'(c) W_{k,t+j} L'_{k,t+j}\Big)$, $k = m, f$, are strictly positive. Thus, unconstrained consumption is strictly increasing in current and future non-labor income, current and future skill prices, and parental human capital, while it is independent of (current and future) prices for home investment goods and child care services. □

Because $E_t = K_t / u'(c_t)$, total investment expenditures are increasing in current consumption, which is increasing in current income levels. Thus, total investment expenditures are increasing in human capital, current skill prices, and current non-labor income (Proposition 1). When households are borrowing constrained, only current income and prices affect investment behavior. By contrast, unconstrained households can efficiently allocate resources across periods, so total investment expenditures are also increasing in all future levels of non-labor income and skill prices. As a consequence, a permanent increase in skill prices will have greater impacts on current investment expenditures (among unconstrained households) than a one-time increase in the price. Additionally, a single period change in wages or non-labor income in period t will have smaller effects on investment that period when constraints are non-binding compared to when they bind. This is not surprising, because any change in income is spread across all periods (in terms of investment and consumption) when families are unconstrained.

Proof of Proposition 1: The proof is immediate from Lemma 1 given that $E_t = K_t/u'(c_t)$ implies

$$\frac{\partial E_t}{\partial \pi} = -K_t \frac{u''(c_t)}{u'(c_t)^2}\frac{\partial c_t}{\partial \pi} \qquad \text{for} \qquad \pi \in \{p_t, P_{c,t}, y_t, w_{m,t}, w_{f,t}, H_m, H_f\}.$$

□

The following corollary shows that increases in the price of household goods inputs or child care lead to reductions in total investment, while an increase in non-labor income raises total investment.

Corollary 1. *Total investment in period t, X_t, is strictly decreasing in the prices of household goods inputs and child care $(p_t, P_{c,t})$, while it is increasing in non-labor income (y_t).*

Proof of Corollary 1: Equation (12) implies that $X_t = \frac{K_t}{\bar{p}_t u'(c_t)}$. Differentiating this with respect to any variable π that affects the composite investment price or consumption implies the following:

$$\frac{\partial X_t}{\partial \pi} = -\frac{X_t}{\bar{p}_t u'(c_t)}\left[\bar{p}_t u''(c_t)\frac{\partial c_t}{\partial \pi} + u'(c_t)\frac{\partial \bar{p}_t}{\partial \pi}\right].$$

Lemma 1 implies that c_t is independent of p_t and P_{ct}, so the fact that $\bar{p}_t$ is increasing in all input prices implies that X_t is decreasing in p_t and P_{ct}. Lemma 1 implies that c_t is increasing in y_t, while $\bar{p}_t$ does not depend on y_t. Together, these imply that X_t is increasing in y_t. □

A.3.3 Responses of Constrained and Unconstrained Families

In this subsection, we compare the responses of constrained and unconstrained single mothers to changes in wages. To simplify the analysis, we assume log utility, $u(c) = \ln(c)$, $v_m(l_m) = \psi_m \ln(l_m)$, with $\psi_m > 0$, and continue to suppose $\beta(1+r) = 1$. Log preferences over consumption imply $E_t = K_t c_t$ (see Equation 12).

Consumption for a constrained mother in period t is given by:

$$c_t^c = \frac{(1+r)A_t + W_{m,t} + y_t - A_{min,t}}{1 + \psi_m + K_t}. \tag{48}$$

If we suppose that the unconstrained mother continues to have the same period utility between $T+1$ and T_R, and we solve her intertemporal problem over the entire horizon, $t...T_R$, she has the lifetime budget constraint:

$$\sum_{j=0}^{T_R-t}(1+r)^{-j}c + \bar{K}_t c = \sum_{j=0}^{T_R-t}(1+r)^{-j}\left[W_{m,t+j}\left(1 - \frac{\psi_m c}{W_{m,t+j}}\right) + y_{t+j}\right] + (1+r)A_t.$$

Solving for c in period t yields

$$c^u = \frac{(1+r)A_t + \sum_{j=0}^{T_R-t}(1+r)^{-j}\left(W_{m,t+j} + y_{t+j}\right)}{\sum_{j=0}^{T_R-t}(1+r)^{-j}\Big(1+\psi_m\Big) + K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}}.$$

Recall that $\bar{K}_t = K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}$.

Constrained mothers only consider current income when making consumption choices in t, while unconstrained mothers take into account discounted future income through retirement at T_R. If the unconstrained mother's horizon ends when her child leaves home ($T_R = T$), and if future expenditures are constant ($\beta\delta_2 = 1$), the denominator in consumption, c^u, for the unconstrained mother simplifies to

$$(1 + \psi_m + K_t) \sum_{j=0}^{T-t} (1 + r)^{-j}.$$

This is the discounted lifetime analogue to the denominator of the constrained mother.

We begin by comparing one time changes in wages in period t. The constrained household responds:

$$\frac{\partial c_t^c}{\partial w_{m,t}} = \frac{H_m}{1 + \psi_m + K_t} > 0.$$

The unconstrained household responds:

$$\frac{\partial c_t^u}{\partial w_{m,t}} = \frac{H_m}{\sum_{j=0}^{T_R - t} (1 + r)^{-j} \Big(1 + \psi_m\Big) + K_t \sum_{j=0}^{T-t} (1 + r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

We can rewrite this as:

$$\frac{\partial c_t^u}{\partial w_{m,t}} = \frac{H_m}{1 + \psi_m + K_t + \sum_{j=1}^{T_R - t} (1 + r)^{-j} (1 + \psi_m) + K_t \sum_{j=1}^{T-t} (1 + r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

So we have,

$$\frac{\partial c_t^c}{\partial w_{m,t}} \geq \frac{\partial c_t^u}{\partial w_{m,t}} > 0.$$

The difference between the constrained and the unconstrained derivatives shrinks as t approaches T. At $t = T$, the derivatives are the same if parent's horizon ends at T ($T = T_R$).

Lemma 2. *A one-time increase in wages, leads to a weakly greater increase in current constrained consumption than current unconstrained consumption.*

Next consider a permanent change in wages. Let $W_{m,t} = \Delta w_{m,t} H_m$, where Δ increases in every period. For constrained mothers, current consumption is not impacted by wages rising in every period:

$$\frac{\partial c_t^c}{\partial \Delta} = \frac{w_{m,t} H_m}{1 + \psi_m + K_t} > 0.$$

The current consumption of unconstrained mothers responds to the wage increasing in every period:

$$\frac{\partial c_t^u}{\partial \Delta} = \frac{\sum_{j=0}^{T_R - t} (1 + r)^{-j} w_{m,t+j} H_m}{\sum_{j=0}^{T_R - t} (1 + r)^{-j} \Big(1 + \psi_m\Big) + K_t \sum_{j=0}^{T-t} (1 + r)^{-j} \left(\beta\delta_2\right)^{-j}} > 0.$$

Here, both the numerator and the denominator of the unconstrained derivative are larger than the constrained derivative.

Lemma 3. *If wages are weakly increasing and $\beta\delta_2 \geq 1$, a permanent increase in wages of Δ, leads to a weakly greater increase in current unconstrained consumption than current constrained consumption. If wages are constant, $\beta\delta_1 = 1$, and $T = T_R$, the responses are the same.*

Proof of Lemma 3 For non-negative wage growth, $w_{t+j} \geq w_t$ for all $j \geq 0$ and

$$
\begin{aligned}
\frac{\partial c_t^u}{\partial \Delta} &= \frac{\sum_{j=0}^{T_R-t}(1+r)^{-j} w_{m,t+j} H_m}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m) + K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}} \\
&\geq \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m) + K_t \sum_{j=0}^{T-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}} \\
&= \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}) - \underbrace{K_t \sum_{j=T-t+1}^{T_R-t}(1+r)^{-j}\left(\beta\delta_2\right)^{-j}}_{=0 \text{ if } T=T_R}} \\
&\geq \frac{w_{m,t} H_m \sum_{j=0}^{T-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}\left(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}\right)}.
\end{aligned}
$$

If $\beta\delta_2 \geq 1$,

$$
\frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\sum_{j=0}^{T_R-t}(1+r)^{-j}\left(1+\psi_m + K_t\left(\beta\delta_2\right)^{-j}\right)} \geq \frac{w_{m,t} H_m \sum_{j=0}^{T_R-t}(1+r)^{-j}}{\left(1+\psi_m + K_t\right)\sum_{j=0}^{T_R-t}(1+r)^{-j}} = \frac{w_{m,t} H_m}{1+\psi_m+K_t} = \frac{\partial c_t^c}{\partial \Delta}.
$$

Notice that if wages are constant, $\beta\delta_2 = 1$, and $T = T_R$, then $\frac{\partial c_t^u}{\partial \Delta} = \frac{\partial c_t^c}{\partial \Delta}$.

A.3.4 Input Quantities

In this subsection, we discuss comparative statics results for input levels, continuing to abstract from uncertainty about wage and income (in the case of unconstrained families). In what follows, we assume log utility, $u(c) = \ln(c)$, $v_m(l_m) = \psi_m \ln(l_m)$, with $\psi_m > 0$, and $\hat{U}((1+r)A_{T+1} + \chi_m H_m) = \chi_0 \ln((1+r)A_{T+1} + \chi_m H_m)$, with $\chi_0 > 0$.[64] To simplify notation, we consider single mother families. The solution for goods investment when families are borrowing constrained is

$$
g_t = \left(\frac{(1+r)A_t + y_t - A_{min,t} + W_{m,t}}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}}\right)\left(\frac{K_t}{1+\psi_m+K_t}\right).
$$

[64] If we assume that the mother has the same log period utility functional forms from $T+1$ to T_R, then $\chi_0 = (1+\psi) \sum_{j=0}^{T_R-T-1}(1+r)^{-j}$

When unconstrained, the solution is

$$g_t = \left(\frac{(1+r)A_t + \sum_{j=0}^{T-t} (1+r)^{-j} \left[W_{m,t+j} + y_{t+j} \right] + (1+r)^{t-T-1} \chi_m H_m}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}} \right) \left(\frac{K_t}{(1+\psi_m)\Upsilon_{T-t} + (1+r)^{t-T-1}\chi_0 + \bar{K}_t} \right).$$

In both cases $\tau_{m,t} = \Phi_{m,t} g_t$ and $Y_{c,t} = \Phi_{c,t} g_t$.

To facilitate the comparative statics analysis below, it is useful to write the problem in a general way such that our results apply equally to both the constrained and unconstrained cases. To that end, we can write g_t in the following general form:

$$g_t = \tilde{K}_t \left(\frac{\tilde{\Omega}_t + \bar{W}_t H_m}{p_t + P_{c,t}\Phi_{c,t} + w_{m,t} H_m \Phi_{m,t}} \right), \tag{49}$$

where we continue to define $D_t \equiv p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}$ (a function of all input prices and H_m). The constant $\tilde{K}_t > 0$ depends on whether constraints are binding or not:

$$\tilde{K}_t = \begin{cases} \frac{K_t}{1+\psi_m+K_t} & \text{if borrowing constrained} \\ \frac{K_t}{(1+\psi_m)\Upsilon_{T-t}+(1+r)^{t-T-1}\chi_0+\bar{K}_t} & \text{if always unconstrained.} \end{cases}$$

The terms collected into $\tilde{\Omega}_t$ and $\bar{W}_t$ will depend on the particular proposition and constrained vs. unconstrained case as discussed below.

Proof of Proposition 2: Here, we consider the effects of changes in $w_{m,t}$ on g_t, $\tau_{m,t}$, and $Y_{c,t}$. We define the $\tilde{\Omega}_t$ and $\bar{W}_t$ terms in Equation (49) as follows:

$$\tilde{\Omega}_t = \begin{cases} (1+r)A_t + y_t - A_{min,t} & \text{if borrowing constrained} \\ (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j} y_{t+j} + (1+r)^{t-T-1}\chi_m H_m + \sum_{j=1}^{T-t}(1+r)^{-j} W_{m,t+j} & \text{if always unconstrained.} \end{cases}$$

and $\bar{W}_t = w_{m,t} > 0$ in both the constrained and always unconstrained cases. Here, $\tilde{\Omega}_t$ reflects all currently available resources not earned from current work and is independent of the prices we consider varying here. As discussed in the text, we assume conditions that ensure $\tilde{\Omega}_t \geq 0$. Here, the conditions are extremely weak in that they only require that the vale of current debt not exceed the present discounted value of all future income (from all sources, including returns on human capital beyond year T).

We now differentiate g_t in Equation (49) with respect to $w_{m,t}$:

$$\frac{\partial g_t}{\partial w_{m,t}} = \tilde{K}_t \left(\frac{D_t H_m - (\tilde{\Omega}_t + w_{m,t} H_m) D_t'}{D_t^2} \right),$$

where D_t' is the derivative of D_t with respect to $w_{m,t}$. Because $D_t H_m > 0$ and $\tilde{\Omega}_t + w_{m,t} H_m \geq 0$, the numerator is strictly positive if $D_t' \leq 0$. Notice

$$D_t' = \frac{(\gamma - \rho) P_{c,t}\Phi_{c,t} a_m \Phi_{m,t}^{\rho}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]} - \frac{\rho H_m \Phi_{m,t}}{1-\rho},$$

which is weakly negative if $\rho \geq \max\{0, \gamma\}$. Therefore, $\frac{\partial g_t}{\partial w_{m,t}} > 0$ if $\rho \geq \max\{0, \gamma\}$, as stated in Proposition 2.

Next, consider the effects of $w_{m,t}$ on $\tau_{m,t}$:

$$\begin{aligned}
\frac{\partial \tau_{m,t}}{\partial w_{m,t}} &= \frac{\partial \Phi_{m,t}}{\partial w_{m,t}} g_t + \frac{\partial g_t}{\partial w_{m,t}} \Phi_{m,t} \\
&= \frac{\Phi_{m,t}\tilde{K}_t}{(1-\rho) w_{m,t} D_t^2} \left\{ \tilde{\Omega}_t [w_{m,t}(\rho - 1)D_t' - D_t] + w_{m,t} H_m [\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t}] \right\}.
\end{aligned}$$

We sign $[w_{m,t}(\rho - 1)D_t' - D_t]$ and $[\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t}]$ separately. First,

$$w_{m,t}(\rho - 1)D_t' - D_t =$$

$$\frac{p_t(1-\gamma)\left[a_m \Phi_{m,t}^{\rho} + a_g\right] + P_{c,t}\Phi_{c,t}[(1-\rho)a_m\Phi_{m,t}^{\rho} + (1-\gamma)a_g] + w_{m,t}H_m\Phi_{m,t}(1-\rho)(1-\gamma)\left[a_m\Phi_{m,t}^{\rho} + a_g\right]}{(\gamma - 1)\left[a_m \Phi_{m,t}^{\rho} + a_g\right]} < 0.$$

Because $\tilde{\Omega}_t \geq 0$, we have $\tilde{\Omega}_t[w_{m,t}(\rho - 1)D_t' - D_t] \leq 0$. Next,

$$\rho(D_t' w_{m,t} - D_t) - D_t' w_{m,t} = \frac{\rho p_t(1-\gamma)\left[a_m\Phi_{m,t}^{\rho} + a_g\right] + P_{c,t}\Phi_{c,t}[\gamma(1-\rho)a_m\Phi_{m,t}^{\rho} + \rho(1-\gamma)a_g]}{(\gamma-1)\left[a_m\Phi_{m,t}^{\rho} + a_g\right]},$$

which is weakly negative if $\min\{\gamma, \rho\} \geq 0$. Therefore, $\frac{\partial \tau_t}{\partial w_{m,t}} < 0$ if $\min\{\gamma, \rho\} \geq 0$ as stated in Proposition 2.

Finally, consider the effects of $w_{m,t}$ on $Y_{c,t}$:

$$\frac{\partial Y_{c,t}}{\partial w_{m,t}} = \frac{\Phi_{c,t}\tilde{K}_t\left\{\tilde{\Omega}_t\Theta_{1,t} + w_{m,t}H_m\Theta_{2,t}\right\}}{w_{m,t}(1-\gamma)(1-\rho)\left[a_m\Phi_{m,t}^{\rho} + a_g\right]D_t^2},$$

where

$$\begin{aligned}
\Theta_{1,t} &= \gamma(1-\rho)a_m\Phi_{m,t}^{\rho}[p_t + w_{m,t}H_m w_{m,t}\Phi_{m,t}] \\
\Theta_{2,t} &= (1-\rho)\left\{a_m\Phi_{m,t}^{\rho}[p_t + (1-\gamma)P_{c,t}\Phi_{c,t} + w_{m,t}H_m\Phi_{m,t}] + (1-\gamma)a_g[p_t + P_{c,t}\Phi_{c,t} + w_{m,t}H_m\Phi_{m,t}]\right\} > 0.
\end{aligned}$$

Clearly, $\frac{\partial Y_{c,t}}{\partial w_{m,t}} > 0$ when $\gamma \geq 0$ as stated in Proposition 2. Also note that if $\tilde{\Omega}_t = 0$ (e.g. no non-labor income and no borrowing/saving), then $\frac{\partial Y_{c,t}}{\partial w_{m,t}} > 0$ holds regardless of γ. $\square$

When families are borrowing constrained, permanent changes in wages have identical effects on behavior as changes in current wages. This is not the case when families are unconstrained; although, the results are the same qualitatively. To see this, define $w_{m,t} = \tilde{w}_{mt}\bar{w}_m$ where $\bar{w}_m$ reflects the permanent component of wages. Now define $\tilde{\Omega}_t$ so that it no longer includes future labor earnings:

$$\tilde{\Omega}_t = (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j} + (1+r)^{t-T-1}\chi_m H_m \geq 0,$$

where the conditions on debt that ensure $\tilde{\Omega}_t \geq 0$ are now stronger than before. (For married couples, $\tilde{\Omega}_t$ would also include the discounted present value of all spousal wages.) All maternal earnings are now included in $\bar{W}_{m,t} = \sum_{j=0}^{T-t}(1+r)^{-j}w_{m,t+j} > 0$. Based on these definitions and Equation (49), the same approach as above shows that all qualitative properties in Proposition 2 apply to permanent changes in wages, $\bar{w}_m$.

The next two propositions consider the effects of parental human capital on specific input quantities. We begin with the case in which parental human capital does not affect the productivity of home goods inputs.

Proposition 7. *Suppose $\bar{\varphi}_g = 0$. Home goods inputs, g, are strictly increasing in H_m and maternal time investment, τ_m, is strictly decreasing in H_m if any of the following conditions are met: (i) $\bar{\varphi}_m < 1$ and $\rho \geq \gamma \geq 0$, (ii) $\bar{\varphi}_m = 1$, or (iii) $\bar{\varphi}_m > 1$ and $\rho \leq \gamma \leq 0$.*

Next, consider $\bar{\varphi}_g > 0$, so the productivity of home goods investment is increasing in maternal human capital. Recall from that the increase in marginal productivity encourages more skilled mothers to shift their investment portfolio towards home goods if inputs are sufficiently substitutable; otherwise, the factor-augmenting nature of H_m can cause them to turn more to other inputs. To focus on the productivity effects of maternal human capital on home goods investment, consider the case of $\bar{\varphi}_m = 1$, which implies equal productivity of H_m at home and in the labor market.

Proposition 8. *Suppose $\bar{\varphi}_m = 1$ (φ_m is CRS) and $\bar{\varphi}_g > 0$. If $\rho \geq \gamma \geq 0$, then home goods investment is strictly increasing in H_m and parental time investment is strictly decreasing in H_m.*

Proofs of Propositions 7 and 8: In Propositions 7 and 8, we study the effects of H_m on input choices. Here, we continue to use the same family resource decomposition as above for constrained families: $\tilde{\Omega}_t = (1+r)A_t + y_t - A_{min,t} \geq 0$ and $\bar{W}_{m,t} = w_{m,t}$. For always unconstrained families, we decompose resources into those related and unrelated to mother's human capital as follows:

$$\begin{aligned}
\tilde{\Omega}_t &= (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}y_{t+j} \geq 0 \\
\bar{W}_{m,t} &= (1+r)^{t-T-1}\chi_m + \sum_{j=0}^{T-t}(1+r)^{-j}w_{m,t+j} > 0,
\end{aligned}$$

where $\tilde{\Omega}_t \geq 0$ now requires our strongest condition on the value of debt (i.e., it cannot exceed the discounted value of all non-labor income). Again, for married couples, $\tilde{\Omega}_t$ would also include the discounted present value of all spousal wages, substantially weakening the condition on debt. The expression $\bar{W}_{m,t}$ corresponds to returns to human capital relevant for the investment decision at time t. For constrained families, it only includes current labor returns, while for unconstrained families, it contains current and all future returns (including the continuation value that depends on maternal human capital).

We denote the derivative of D_t with respect to maternal human capital by $D_t' = P_{c,t}\frac{\partial \Phi_{c,t}}{\partial H_m} + w_{m,t}\Phi_{m,t} + w_{m,t}H_m\frac{\partial \Phi_{m,t}}{\partial H_m}$. Consider the effects of changes in H_m on g_t by differentiating Equation (49):

$$\frac{\partial g_t}{\partial H_m} = \tilde{K}_t\left(\frac{D_t\bar{W}_{m,t} - (\tilde{\Omega}_t + \bar{W}_{m,t}H_m)D_t'}{D_t^2}\right),$$

which is positive if $D_t' \leq 0$. Notice

$$D_t' =$$

$$\frac{P_{c,t}\Phi_{c,t}\left\{a_m\Phi_{m,t}^{\rho}\left[(\gamma-\rho)(1-\bar{\varphi}_m)+\rho(\gamma-1)\bar{\varphi}_g\right]+\bar{\varphi}_g(\rho-1)\gamma a_g\right\}+w_{m,t}H_m\Phi_{m,t}\rho(\gamma-1)(\bar{\varphi}_g+1-\bar{\varphi}_m)[a_m\Phi_{m,t}^{\rho}+a_g]}{H_m(1-\rho)(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]}$$

We see that $D_t' \leq 0$, and therefore $\frac{\partial g_t}{\partial H_m} > 0$ if $(\rho-\gamma)(1-\bar{\varphi}_m)+\rho(1-\gamma)\bar{\varphi}_g \geq 0$, $\gamma\bar{\varphi}_g \geq 0$, and $\rho(\bar{\varphi}_g+1-\bar{\varphi}_m) \geq 0$.

When $\bar{\varphi}_g = 0$, we have $(\rho-\gamma)(1-\bar{\varphi}_m) \geq 0$ and $\rho(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\gamma \geq 0$ (Proposition 8).

Next, consider maternal time investment:

$$\begin{aligned}\frac{\partial \tau_{m,t}}{\partial H_m} &= \Phi_{m,t}\frac{\partial g_t}{\partial H_m}+\frac{\partial \Phi_{m,t}}{\partial H_m}g_t \\ &= \frac{\Phi_{m,t}\tilde{K}_t}{D_t^2 H_m(1-\rho)}\left[\bar{W}_{m,t}H_m\big(\rho(\bar{\varphi}_m-1-\bar{\varphi}_g)D_t+(\rho-1)D_t'H_m\big)+\tilde{\Omega}_t\big((\rho(\bar{\varphi}_m-\bar{\varphi}_g)-1)D_t+(\rho-1)D_t'H_m\big)\right].\end{aligned}$$

We have two parts of this expression to sign. First:

$$\bar{W}_{m,t}H_m\left\{\rho(\bar{\varphi}_m-1-\bar{\varphi}_g)D_t+(\rho-1)D_t'H_m\right\} = \left[\frac{1}{(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]}\right]\Big\{p_t\rho(\bar{\varphi}_m-\bar{\varphi}_g-1)(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]+$$
$$P_{c,t}\Phi_{c,t}\left[a_m\Phi_{m,t}^{\rho}\gamma(1-\rho)(\bar{\varphi}_m-1)+a_g[(\gamma-\rho)\bar{\varphi}_g+\rho(1-\gamma)(\bar{\varphi}_m-1)]\right]\Big\},$$

which is positive when: $\rho(\bar{\varphi}_m-\bar{\varphi}_g-1) \geq 0$, $\gamma(\bar{\varphi}_m-1) \geq 0$, and $(\gamma-\rho)\bar{\varphi}_g+\rho(1-\gamma)(\bar{\varphi}_m-1) \geq 0$. It is negative when: $\rho(\bar{\varphi}_g+1-\bar{\varphi}_m) \geq 0$, $\gamma(1-\bar{\varphi}_m) \geq 0$, and $(\rho-\gamma)\bar{\varphi}_g+\rho(1-\gamma)(1-\bar{\varphi}_m) \geq 0$.

Second:

$$\tilde{\Omega}_t\left\{(\rho(\bar{\varphi}_m-\bar{\varphi}_g)-1)D_t+(\rho-1)D_t'H_m\right\} = \left[\frac{1}{(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]}\right]\Big\{w_{m,t}H_m\Phi_{m,t}(\rho-1)(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]+$$
$$p_t(\rho(\bar{\varphi}_m-\bar{\varphi}_g)-1)(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g]+P_{c,t}\Phi_{c,t}\left[a_m\Phi_{m,t}^{\rho}(1-\rho)(\gamma\bar{\varphi}_m-1)+a_g[(\gamma-\rho)\bar{\varphi}_g+(1-\gamma)(\rho\bar{\varphi}_m-1)]\right]\Big\}.$$

Because the first part of the expression in braces $w_{m,t}H_m\Phi_{m,t}(\rho-1)(1-\gamma)[a_m\Phi_{m,t}^{\rho}+a_g] < 0$, there is always a negative force (independent of parameters) impacting the effect of mother's human capital on time investment when $\tilde{\Omega}_t > 0$. We can only give cases where the derivative is (strictly) decreasing in mother's human capital. The entire expression related to $\tilde{\Omega}_t$ is negative when: $(1-\gamma)(1-\rho\bar{\varphi}_m)+\bar{\varphi}_g(\rho-\gamma) \geq 0$, $1-\gamma\bar{\varphi}_m \geq 0$, and $1+\rho(\bar{\varphi}_g-\bar{\varphi}_m) \geq 0$.

Altogether, conditions that imply a strictly negative (when $\tilde{\Omega}_t > 0$) impact of maternal human capital on time investment are as follows:

1. $\rho+\rho(\bar{\varphi}_g-\bar{\varphi}_m) \geq 0$,
2. $\gamma-\gamma\bar{\varphi}_m \geq 0$,
3. $(1-\gamma)\rho(1-\bar{\varphi}_m)+\bar{\varphi}_g(\rho-\gamma) \geq 0$,
4. $(1-\gamma)(1-\rho\bar{\varphi}_m)+\bar{\varphi}_g(\rho-\gamma) \geq 0$,
5. $1-\gamma\bar{\varphi}_m \geq 0$,
6. $1+\rho(\bar{\varphi}_g-\bar{\varphi}_m) \geq 0$.

Note that condition 1 implies condition 6, condition 2 implies condition 5, and condition 3 implies condition 4. We are left with conditions 1–3. When $\bar{\varphi}_g = 0$, we have $\rho(1-\bar{\varphi}_m) \geq 0$ and $\gamma(1-\bar{\varphi}_m) \geq 0$ (Proposition 7). And, when $\bar{\varphi}_g > 0$ and $\bar{\varphi}_m = 1$, we have $\rho \geq 0$ and $\rho \geq \gamma$ (Proposition 8). □

A.3.5 Closed form expressions for total investment

If we follow Del Boca, Flinn, and Wiswall (2014) (and several subsequent papers) by assuming log preferences for consumption and leisure (i.e., $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, $\psi_j \geq 0$, for $j \in \{m, f\}$), then we obtain a closed form expression for total investment among constrained households:

$$X_t = \frac{K_t \left[(1+r)A_t + W_{m,t} + W_{f,t} + y_t - A_{min,t}\right]}{\bar{p}_t \left[1 + \psi_m + \psi_f + K_t\right]}. \tag{50}$$

From this, we see that the dynamics of constrained investment depend on both the dynamics of input prices through $\bar{p}_t$ and the dynamics of "full" family income, $W_{m,t} + W_{f,t} + y_t$.

If we also assume a log continuation utility (i.e., $\tilde{U}(H_m, H_f, A) = \chi_0 \ln(A + \chi_m H_m + \chi_f H_f)$, with χ_0, χ_m, and χ_f all non-negative), then we obtain a very similar closed form expression for total investment in the unconstrained case:

$$X_t = \frac{K_t \left[(1+r)A_t + \sum_{j=0}^{T-t} (1+r)^{-j} \left(W_{m,t+j} + W_{f,t+j} + y_{t+j}\right) + (1+r)^{-(T+1-t)} \left(\chi_m H_m + \chi_f H_f\right)\right]}{\bar{p}_t \left[(1+\psi_m+\psi_f)\Upsilon_{T-t} + (1+r)^{-(T+1-t)}\chi_0 + \bar{K}_t\right]}. \tag{51}$$

Total investment for unconstrained families depends on the discounted present value of lifetime (rather than current) "full" income as well as the continuation value of parental human capital. Also, note that the denominator reflects discounted lifetime sums of $(1+\psi_m+\psi_f)$ and K_t rather than only their current values. As a result, a single period change in wages or non-labor income in period t will have much smaller effects on investment that period when constraints are not binding compared to when they bind.

A.4 Effects of a Small Price Change

Proof of Proposition 3: From Equation (10), the price elasticity of total investment is

$$\frac{\partial \ln X_t}{\partial \ln \pi_t} = -\frac{\partial \ln \overline{p}_t}{\partial \ln \pi_t} - \frac{\partial \ln u'(c_t)}{\partial \ln \pi_t}, \quad \forall \pi_t \in \Pi_t.$$

First, we show that the second term $\partial \ln u'(c_t)/\partial \ln \pi_t$ does not depend on the within-period production function. Lemma 1 implies that $\partial \ln u'(c_t)/\partial \ln p_t = \partial \ln u'(c_t)/\partial \ln P_{c,t} = 0$. Parental wages $W_{m,t}$ and $W_{f,t}$ affect consumption and leisure decisions through the budget constraint. But $\overline{p}_t$ does not play any role in shaping the relationship between parental wages and consumption/leisure choices, so $\partial \ln u'(c_t)/\partial \ln W_{m,t}$ and $\partial \ln u'(c_t)/\partial \ln W_{f,t}$ do not depend on the within-period production function.

Next, we show that the first term $\partial \ln \overline{p}_t/\partial \ln \pi_t$ depends on the within-period production function only through input expenditure shares. Notice that the composite price can be written as the minimum unit cost of production:

$$\overline{p}_t(\Pi_t) = \min_{\tau_{m,t},\tau_{f,t},g_t,Y_{c,t}} \left\{ W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + p_t g_t + P_{c,t}Y_{c,t} | f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}) \geq 1 \right\}.$$

Let $(\underline{\tau}_{m,t}(\Pi_t), \underline{\tau}_{f,t}(\Pi_t), \underline{g}_t(\Pi_t), \underline{Y}_{c,t}(\Pi_t))$ be the solution to this problem. Then, by application of the envelope theorem (Shephard's Lemma), we have

$$\frac{\partial \overline{p}_t(\Pi_t)}{\partial p_t} = \underline{g}_t(\Pi_t).$$

Therefore, the elasticity of $\overline{p}_t$ with respect to p_t is

$$\frac{\partial \ln \overline{p}_t(\Pi_t)}{\partial \ln p_t} = \frac{p_t \underline{g}_t(\Pi_t)}{\overline{p}_t(\Pi_t)} = \frac{p_t \underline{g}_t(\Pi_t) X_t}{\overline{p}_t(\Pi_t) X_t} = S_{g,t}(\Pi_t).$$

This holds for all input prices. That is,

$$\frac{\partial \ln \overline{p}_t(\Pi_t)}{\partial \ln \pi_t} = S_{\pi,t}(\Pi_t), \quad \forall \pi_t \in \Pi_t.$$

B Time Investment Categories in PSID-CDS

This appendix lists all the activities we include in our parental time investment measure when children are actively engaged with their mother and/or father. This list is based in the 1997 coding, but the categories are very similar in 2002 and 2007.

B.1 Academic investment activities

1. PASSIVE LEISURE - Reading
   - 939 Reading or looking at books; 941 Reading magazines, reviews, pamphlets; 959 Reading newspapers; 942 Reading, NA what; 943 Being read to, listening to a story;979 Letters (reading or writing) and reading mail)
2. EDUCATIONAL AND PROFESSIONAL TRAINING
   - 519 Other classes, courses, lectures, academic or professional if the child is not a full-time student or NA whether a student, being tutored; 549 Homework (non-computer related), studying, research, reading, related to classes or profession; 569 Other education; "watched a slide program";
3. HOME COMPUTER RELATED ACTIVITIES
   - 501 Lessons in computers (Learning how to use a computer); 504 Using the computer for homework, studying, research, reading related to classes or profession, except for current job; 510 Media, reading the newspaper, stock quotes, weather reports; 511 Library functions (using computer/internet to acquire specialized information); 512 Computer work, getting computer programs to work, reading the manual, repairing computer, setting up computer;

B.2 Health investment activities

1. SERVICES
   - 339 Medical care for self; visits to doctor, dentist, optometrist, including making appointments
2. CARE TO SELF
   - 411 Medical care at home to self; taking care of own sickness.
3. OTHER PERSONAL AND HELPING
   - 488 Receiving child care, a child is the passive recipient of personal care, medical care from parent or other, baby being held, being comforted by a parent

B.3 Play

1. PLAYING/GAMES
    - 866-889 all subcategories

B.4 Arts and crafts with household children

1. HOBBIES
    - 831-835 all subcategories
2. DOMESTIC CRAFTS
    - 841-844 all subcategories
3. ART AND LITERATURE
    - 851-852 all subcategories
4. MUSIC/THEATER/DANCE
    - 861-864 all subcategories
5. CLASSES/LESSONS FOR LEISURE ACTIVITY
    - 887 Lessons in music, singing, instruments

B.5 Sports

1. CLASSES/LESSONS FOR LEISURE ACTIVITY
    - 881 Lessons in dance; 885 Lessons in sports activities such as swimming, golf, tennis, skating, roller skating; 886 Lessons in gymnastics, yoga, judo, body movement; 888 Other lessons, not listed above
2. COMPETITIVE SPORTS-OTHER EDUCATIONAL ACTIVITIES
    - 883 Organized meets, games, practices for team sports
3. ACTIVE LEISURE ACTIVITIES
    - 884 Meets, practices for individual sports
4. ACTIVE SPORTS
    - 801-810, 865 all subcategories
5. OTHER OUT OF DOORS
    - 811-818, 824, 825, 826 all subcategories
6. WALKING
    - 821-823 all subcategories

B.6 Talk and listen

1. PASSIVE LEISURE

   - 962 Other talking/conversations face-to-face conversations, mixed or non-household people in conversation; 963 Conversations with other household members–adults and/or children; 967 Receiving instructions, orders

B.7 Eating

1. INDOOR

   - 108-119 Meal preparation activities

2. CARE TO SELF

   - 439 Meals at home; including coffee, drinking, food from a restaurant eaten at home; 448 Meals away from home eaten at a friend's/relative's home; 449 Meals away from home - eating at restaurants

B.8 Socializing

1. CHILD CARE FOR OTHER HOUSEHOLD CHILDREN

   - 221 Helping children learn (fix things, bake cookies, etc.); 222 Help with homework or supervising homework; 238 Reading to a child; 239 Conversations with or listening to household children only in the context of child care arrangement; 248 Playing with household babies ages 0-2, "playing with baby", indoors or outdoors; 249 Respondents playing indoors with children; 258 Coaching/leading outdoors/non-organizational activities; 259 Respondents playing outdoors with children

2. CARE TO SELF

   - 484 Affection between household and non-household members; giving and getting hugs, kisses, sitting on laps

3. SOCIALIZING

   - 752-799 all subcategories

4. PROFESSIONAL/UNION ORGANIZATIONS

   - 601-602 all subcategories

5. CHILD/YOUTH/FAMILY ORGANIZATIONS

   - 671-672 all subcategories

6. FRATERNAL ORGANIZATIONS

   - 661-662 all subcategories

7. POLITICAL PARTY AND CIVIC PARTICIPATION
    - 621-622 all subcategories
8. SPECIAL INTEREST/IDENTITY ORGANIZATIONS
    - 611-612 all subcategories
9. OTHER MISCELLANEOUS ORGANIZATIONS
    - 689 Other organizations; any activities of an organization not fitting into the above categories
    - 698- 699 Related travel

B.9 Religious activities

1. RELIGIOUS PRACTICE
    - 651-652 all subcategories
2. RELIGIOUS GROUPS
    - 641-644 all subcategories

B.10 Volunteering

1. VOLUNTEER, HELPING ORGANIZATIONS
    - 631-635 all subcategories

B.11 Other activities

1. SERVICES
    - 377 Other professional services; lawyer, counseling (therapy).
2. TRAVEL
    - 597-599 School-related travel; 899 Related travel to sports/active leisure; waiting for related travel; vacation travel
3. ATTENDING SPECTACLES, EVENTS
    - 709-749 all subcategories

C Additional Data Sources

C.1 Child Care Prices

Child care costs for 4-year old center-based care, P_c, are obtained from annual reports on the cost of child care in the U.S. compiled by Child Care Aware of America (2009–2019).[65] These costs represent the average annual price charged by full-time center providers in each state covering 2006 to 2018. Several values from annual reports were dropped if they were imputed based on previous survey years or were taken from different sources or subsets of locations.

In order to obtain child care cost measures going back to 1997, we use our data (from 2006–2018) to regress state-year child care costs on state fixed effects, a linear time trend, and average state-year hourly wages for child care workers. The estimated coefficient on the linear time trend is 217.5, while the coefficient on average wages for child care workers is 18.8. The state-fixed effects explain most of the variation, and the R^2 statistic for this regression is 0.89. (Regressing the child care price on average wages for child care workers, without state or year fixed effects, yields a coefficient of 576.9 and R^2 statistic of 0.28.) Average wages for child care workers are estimated from the 1992–2019 monthly *Current Population Surveys (CPS)*.[66] We then use the estimated coefficients, including the state fixed effects, to impute child care costs back to 1997 (or for any missing observations) using *CPS* average wages for child care workers for each state and year.

Finally, to put child care prices in roughly hourly terms, consistent with our parental wage measures, we divide our child care cost measures by 33×52, reflecting an average of 33 hours per week spent in family- or center-based child care among young children of employed mothers (Laughlin 2013).

C.2 Household Input Prices

We obtain state-year measures of household-based goods input prices, p, from a combination of goods and services price series from the *Regional Price Parities by State (RPP)* from the U.S. Bureau of Economic Analysis (BEA) and the *Consumer Price Index (CPI-U)* from the U.S. Bureau of Labor Statistics (BLS). The *RPP*'s measure price level differences relative to the U.S. average by state and are available from 2008 to 2017. These indices are divided into several categories: All items; Goods; Services: Rent; and, Other Services.

To create the goods price series by state, we take the U.S. average of the *CPI* for "Commodities" and multiply it by each state's "Goods" *RPP*. This produces price measures by state for 2008–2017. To project back to 1997, we take the regional *CPI* for "Commodities" and use the year-over-year change of this index for each state within its Census region (Northeast, Midwest, South and West), working back from 2008 values. To create the services price series, we follow the same steps, using the "Services: Other" component from the *RPP*'s and the "Services less rent of shelter" index from the CPI. All these prices are year averages using a base year of 2000.

[65] We are grateful to Kristina Haynie of Child Care Aware of America for providing us with a digital compendium of child care prices from all annual reports. Each year, states report the annual prices that child care providers charge for their services. These reports are provided by Child Care Resource and Referral (CCR&R) agencies in each state.

[66] We restrict our *CPS* sample to workers who are at least 18 years old, report either weekly earnings or an hourly wage, and report an occupation of either child care worker or preschool or kindergarten teacher (2010 occupation classification codes 4600 or 2300). Among workers reporting weekly earnings, an hourly wage is calculated from weekly earnings divided by usual hours worked per week. CPS weights are used to calculate state-year average wages.

Finally, we use as our household goods input price, p, a weighted average of these goods and services price series, with a weight of 0.3 on services, reflecting the greater share of goods in the bundle of child investment inputs. For example, we use the 2003–18 *Consumer Expenditure Survey (CEX)* to create a comprehensive measure of potential household investments in children that includes expenditures on "goods" and "services" as described in Appendix C.3 and Appendix Table C-1. Based on this comprehensive measure of household investment inputs, we find that families with 1–2 children, both ages 0–12, spend an average of 35% of all household investment dollars on services. Taking a more limited household investment measure closer to that used in our *PSID-CDS* analysis suggests that families spend, on average, 20% on service-related child investments.

C.3 Consumer Expenditure Survey

The *Consumer Expenditure Survey (CEX)* is a rotating panel gathered by the U.S. Bureau of Labor Statistics. It collects detailed information on consumption, income and household's characteristics, and is representative of the U.S. population. The unit of measurement for the survey is given by Consumer Units. These units are broadly defined as members of a household that are related, or two or more persons living together that use their incomes to make joint expenditures decisions. Each unit is interviewed for up to four times during a 12-month period and is asked to report their expenditures on a detailed set of categories for the preceding three months. After completing the four interviews, each consumer unit is replaced.

The sample we use runs from 2003 to 2018. We exclude consumer units that do not complete all four interviews and those whose key characteristics are inconsistent over time (i.e., changes in age or race of the reference person, or if the family size changes by more than two members), indicating a likely change in families in the unit. We limit our sample to families with parents ages 18–65, mothers who were ages 16–45 when their youngest child was born, and with only 1–2 children (all age 12 or younger).

We use the Universal Classification Codes (UCCs) for expenditure categories to create our household-level investment measures. Our preferred investment measure is composed of two broad categories: investment in goods and in services. Investment in goods includes expenditures on books (for school or other, magazines, etc.), toys, games, musical instruments, and other learning equipment such as computers and accessories for nonbusiness use. The services measure includes admission fees for recreational activities, fees for recreational lessons and tutoring services. We sum the quarterly expenditures reported by each household (across categories and their four interviews) to obtain annual investment measures, then divide by 52 to create weekly measures.

Table C-1 provides a more detailed look at the expenditure categories, along with their average weekly expenditures.[67] We also report household investment expenditure categories consistent with those collected by the *PSID-CDS*. Altogether, the *PSID-CDS* categories aggregate to a weekly expenditure amount of $585.25, roughly 60% of the spending we include from the *CEX*.

[67]We aggregate a few categories, because some categories split over time.

Table C-1: Household Investment Expenditure Categories and Average Weekly Expenditures in the CEX

UCC	Description	PSID CDS	Average Expenditure (2002 dollars)
	Goods:		**561.75**
590220	-Books through book clubs	X	4.41
590230	-Books not through book clubs	X	43.00
590310	-Magazine or newspaper subscription		17.06
590410	-Magazine or newspaper, single copy		6.38
610110	-Toys, games, arts, crafts, tricycles, and battery powered riders	X	203.71
610120	-Playground equipment	X	10.89
610130	-Musical instruments, supplies, and accessories		26.02
660210	-School books, supplies, equipment for elementary, high school	X	24.36
660310	-Encyclopedia and other sets of reference books	X	0.31
660900, 660901	-School books, supplies, equipment for day care, nursery, preschool.	X	2.63
660902	-School books, supplies, and equipment for other schools	X	1.71
660410	-School books, supplies, equipment for vocational and technical schools	X	0.51
670902	-Other school expenses including rentals	X	47.61
690111	-Computers and computer hardware for nonbusiness use		134.65
690112, 690119, 690120	-Computer software and accessories for non-business use		22.48
690117	-Portable memory		2.88
690118	-Digital book readers	X	10.72
690230	-Business equipment for home use		2.43
	Services:		**421.09**
620211, 620212, 620213, 620214, 620215, 620216	-Admission fees for entertainment activities, including movie, theater, concert, opera or other musical series (single admissions and season tickets)		179.22
620310	-Fees for recreational lessons or other instructions	X	223.87
620904	-Rental and repair of musical instruments, supplies, and accessories		2.56
670903	-Test preparation, tutoring services	X	11.53
690113	-Repair of computer systems for nonbusiness use		3.92
	Total Investment		**982.85**

D Details on Counterfactual Analysis

D.1 No Borrowing/Saving

Our main counterfactual analysis assumes that parents have log preferences for consumption and leisure and are borrowing constrained. These assumptions permit a closed form solution for total investment. See Equation (50). We further assume that parents have no non-labor income and cannot borrow or save ($y_t = A_t = A_{min,t} = 0$). Their subjective discounter factor is $\beta = 1/1.02$ and they value their children's achievement at age 13 ($T = 13$). Finally, individuals are endowed with 100 hours per week (5,200 hours per year), which they can use for market work, leisure, or time investment in children.

These assumptions, along with estimated technology parameters and calibrated preference parameters, allow us to simulate investment and achievement for each child in 2002 PSID.

D.1.1 Calibration of Preference Parameters

The utility weights of the Cobb-Douglas utility function (α, ψ_m, and ψ_f) determine how households allocate their resources between consumption, leisure, and child investment in each period. For example, Equation (50) shows that two-parent households spend a fraction $K_t/(1 + \psi_m + \psi_f + K_t)$ of their full income on total investment in children. Therefore, given prices and technology parameters, the preference parameters can be identified from the levels of parental time spent on market work and child investment. We choose the preference parameters so that the model replicates weekly time use patterns from the 2002 PSID.

Table D-1: Calibration Targets: Weekly Hours of Time Investment and Work

	Mother's Education	
	Non-College	College
A. Single Mothers		
Mother's Time Investment	10.04	12.42
Mother's Hours Worked	42.26	38.22
B. Two-Parent Households		
Mother's Time Investment	9.56	12.13
Mother's Hours Worked	38.43	38.58
Father's Hours Worked	43.85	44.03

Table D-2: Calibrated Preference Parameters (No Borrowing/Saving)

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	3.93	4.90
ψ_m	1.27	1.46
B. Two-Parent Households		
α	2.26	3.11
ψ_m	0.50	0.54
ψ_f	0.66	0.57

Tables D-1 and D-2 show calibration targets and calibrated parameters, separately by marital status and mother's education (non-college vs. college). The calibrated parameters imply that college-educated mothers have a stronger preference for their child's skills (α) compared to non-college-educated mothers. College educated single mothers have a lower value of leisure than their non-college counterparts, while the opposite is true for married mothers. College educated fathers have a lower value of leisure than non-college fathers.

D.1.2 Additional Counterfactual Simulation Results

Table D-3 reports the percentage change in total investment (at age 5), X_t, in response to input price changes of different magnitudes. Consistent with Proposition 3, responses to small price changes are very similar for the nested CES and Cobb-Douglas specifications, but the differences grow with larger price changes.

D.2 Unconstrained

In this subsection, we provide additional counterfactual analysis without binding borrowing constraints. We continue to assume $u(c) = \ln c$, $v_j(l_j) = \psi_j \ln l_j$ for $j \in \{m, f\}$, $\beta = 1/1.02$, and $y_t = 0$ for all t. In addition, we make assumptions specific to the unconstrained case. As in Appendix A.3.3, we assume that $\beta(1+r) = 1$, parents continue to have the same period utility after period T, and work until period T_R. They live until period T_D and have zero assets at the time of the child's birth, $A_1 = 0$.

D.2.1 Model solution

As of $t = 1$, the lifetime budget constraint for single mothers is

$$\sum_{t=1}^{T_D}(1+r)^{-t+1}c_t + \sum_{t=1}^{T}(1+r)^{-t+1}E_t = \sum_{t=1}^{T_R}(1+r)^{-t+1}W_{m,t}(1-l_{m,t}). \tag{52}$$

Using the optimality conditions for investment and leisure, Equations (10) and (26), and the fact that consumption is constant due to $\beta(1+r) = 1$, we can back out the period consumption from the lifetime budget constraint (52):

$$c = \left[\frac{1-(1+r)^{-T_D}}{1-(1+r)^{-1}} + \frac{1-\delta_2^{-T}}{1-\delta_2^{-1}}\alpha\beta^T\delta_2^{T-1}\delta_1 + \frac{1-(1+r)^{-T_R}}{1-(1+r)^{-1}}\psi_m\right]^{-1}\left[\sum_{t=1}^{T_R}(1+r)^{-t+1}W_{m,t}\right]. \tag{53}$$

From c given by (53), we can calculate investment and leisure and using the optimality conditions (10) and (26).

Because we assume zero asset at child birth ($A_1 = 0$), we can calculate the asset level at child age 5 (A_5) using the period budget constraint as follows:

$$A_{t+1} = (1+r)A_t + W_{m,t}(1-l_{m,t}) - c_t - E_t.$$

For price-reduction simulations, we assume that the price reduction (for all future periods $t \geq 5$) occurs unexpectedly at child age 5 ($t = 5$), in which case we solve parents' problem as of period $t = 5$, taking

Table D-3: Elasticity of Total Investment Quantity with Respect to Input Prices (No Borrowing/Saving)

Price Change	Nested CES				Cobb-Douglas				% Difference between Cobb-Douglas and Nested CES			
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care
A. Single Mothers												
10% Change	0.28	-0.80	-0.04	-0.23	0.28	-0.80	-0.05	-0.24	0.37	-0.14	6.20	4.86
30% Change	0.32	-0.97	-0.05	-0.25	0.31	-0.99	-0.05	-0.28	-4.35	2.04	16.56	10.34
50% Change	0.38	-1.24	-0.05	-0.29	0.34	-1.32	-0.06	-0.34	-9.78	5.95	31.80	18.62
B. Two-Parent Households												
10% Change	0.16	-0.93	-0.03	-0.13	0.16	-0.94	-0.03	-0.13	-2.65	0.51	4.39	2.02
30% Change	0.19	-1.16	-0.03	-0.14	0.17	-1.18	-0.03	-0.15	-8.07	1.88	14.80	7.81
50% Change	0.23	-1.54	-0.03	-0.15	0.20	-1.60	-0.04	-0.18	-14.57	4.39	29.96	16.29

A_5 and new prices as given. From the lifetime budget constraint as of $t = 5$, the new level of period consumption after the price reduction is calculated as follows:

$$c = \left[\frac{1-(1+r)^{-(T_D-4)}}{1-(1+r)^{-1}} + \frac{1-\delta_2^{-(T-4)}}{1-\delta_2^{-1}}\alpha\beta^{T-4}\delta_2^{T-5}\delta_1 + \frac{1-(1+r)^{-(T_R-4)}}{1-(1+r)^{-1}}\psi_m\right]^{-1} \times \left[(1+r)A_5 + \sum_{t=5}^{T_R}(1+r)^{-t+5}W_{m,t}\right].$$

Notice that only wage reduction affects the consumption level.

D.2.2 Calibration of Preference Parameters

We assume parents work until age 65 and live until age 80 (based on the average age for two-parent households). Since we do not observe parents' wages over their entire career, we use estimated life-cycle profile of wages, which we construct in the following way. First, using data from PSID, separately for mothers and fathers, we regress log hourly wages on potential experience and experience squared, state and year dummies, and individual fixed effects. Let $\mathcal{W}_j(x_{j,t})$ be the wage of a parent $j \in \{m, f\}$ predicted by their potential experience $x_{j,t}$ in year t.

Next, we construct future and past wages based on the wage in 2002 and predicted wages:

$$W_{j,t} = W_{j,2002}\frac{\mathcal{W}_j(x_{j,t})}{\mathcal{W}_j(x_{j,2002})}, \quad \forall t \neq 2002.$$

Notice that this approach assumes that the gap between actual and predicted wage in 2002 reflects individual fixed effects.

As before, we calibrate the preference parameters (α, ψ_m, ψ_f) seprately by marital status and maternal education group by targeting average time spent on investment and market work (presented in Table D-1). The calibrated parameters for the unconstrained case, shown in Table D-4, exhibit patterns that are qualitatively similar to those of the constrained case.

Table D-4: Calibrated Preference Parameters (Unconstrained)

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	7.63	8.17
ψ_m	1.63	1.62
B. Two-Parent Households		
α	4.13	5.99
ψ_m	0.65	0.69
ψ_f	0.76	0.72

D.2.3 Counterfactual Simulations

Tables D-5 and D-6 report counterfactual simulations analogous to Tables 6 and 7 under the assumption that families are unconstrained.

Table D-5: Gaps in Investment (% Difference) between Non-College and College (Unconstrained)

	Baseline	Equalizing:				
		Wages	All Prices	Technology	All Prices and Technology	Preferences
A. Single Mothers						
Total Investment						
Expenditure (E)	49.47	13.64	13.64	49.47	13.64	39.57
Price ($\bar{p}$)	14.12	-7.44	-3.20	18.98	-1.41	14.12
Quantity (X)	32.26	22.08	17.65	28.53	15.24	23.50
Mother's Time Investment (τ_m)	23.75	18.23	15.67	22.78	15.48	15.55
B. Two-Parent Households						
Total Investment						
Expenditure (E)	107.46	40.83	40.83	107.46	40.83	44.58
Price ($\bar{p}$)	49.70	7.56	6.08	50.18	4.48	49.70
Quantity (X)	36.63	30.38	32.62	36.14	34.14	-4.79
Mother's Time Investment (τ_m)	26.97	26.28	27.40	32.54	34.43	-11.53

Table D-6: Effects of 30% Reduction in Prices (Unconstrained)

	Nested CES					Cobb-Douglas				
	Wages (ages 5+)	Wages (ages 5–12)	Wages (Constant income)	Goods	Child Care	Wages (ages 5+)	Wages (ages 5–12)	Wages (Constant income)	Goods	Child Care
A. Single Mothers										
Change in Investment at Age 5 (%):										
Total Expenditure (E)	-29.92	-18.48	0.00	0.00	0.00	-29.92	-18.48	0.00	0.00	0.00
Mother's Time (τ_m)	-5.74	9.95	34.51	1.29	3.92	0.10	16.89	42.86	0.00	0.00
Goods (g)	-11.97	2.51	25.62	8.65	3.75	-29.91	-18.48	0.00	42.86	0.00
Childcare (Y_c)	-20.09	-6.98	14.02	0.71	23.52	-29.90	-18.55	0.00	0.00	42.86
Total (X)	-9.67	5.29	28.89	1.42	7.72	-9.20	5.87	29.56	1.65	8.49
Effects on Age 13 Achievement:										
100×Log Achievement (Age 13)	-5.40	2.13	12.47	1.21	3.45	-4.99	2.53	12.88	1.38	3.71
Value (% Cons. over Ages 5–12)	-5.71	2.37	14.59	1.33	3.82	-5.29	2.83	15.10	1.52	4.11
B. Two-Parent Households										
Change in Investment at Age 5 (%):										
Total Expenditure (E)	-29.92	-18.88	0.00	0.00	0.00	-29.92	-18.88	0.00	0.00	0.00
Mother's Time (τ_m)	-3.28	12.42	38.05	0.76	2.10	0.08	16.34	42.85	0.00	0.00
Father's Time (τ_f)	-3.19	12.41	38.17	0.76	2.04	0.11	16.21	42.86	0.00	0.00
Goods (g)	-9.70	4.70	28.87	8.10	1.98	-29.93	-18.81	0.00	42.85	0.00
Childcare (Y_c)	-18.42	-5.10	16.45	0.45	21.55	-29.93	-18.65	0.00	0.00	42.86
Total (X)	-5.63	9.68	34.70	0.91	4.15	-5.16	10.19	35.36	1.03	4.47
Effects on Age 13 Achievement:										
100×Log Achievement (Age 13)	-2.99	4.43	14.89	0.74	1.81	-3.35	4.07	14.53	0.23	1.36
Value (% Cons. over Ages 5–12)	-2.15	3.31	11.74	0.54	1.33	-2.72	2.70	11.09	-0.14	0.67

D.3 Free Child Care Policies

Consider a policy that gives a certain amount of child care, denoted by $\overline{Y}_{c,t}$, for free. In this case, households' out-of-pocket child care expenditure is a non-linear function of total child care investment $Y_{c,t}$:

$$\max\left\{P_{c,t}(Y_{c,t}-\overline{Y}_{c,t}),0\right\}$$

As a result, the total investment expenditure, $\mathcal{E}_t(X_t)$, also depends on total investment X_t non-linearly. For single mothers, it is given by

$$\mathcal{E}_t(X_t;\Pi_t,\overline{Y}_{c,t})=\min_{\tau_{m,t},g_t,Y_{c,t}}\left\{W_{m,t}\tau_{m,t}+p_tg_t+\max\left\{P_{c,t}(Y_{c,t}-\overline{Y}_{c,t}),0\right\}\mid f_t(\tau_{m,t},g_t,Y_{c,t})\geq X_t\right\}.$$

Let $X_{H,t}(X_t,Y_{c,t})$ be the amount of composite home investment (see Section 3.1) that is required to produce X_t for a given level of child care $Y_{c,t}$. Since the expenditure on home investment is still a linear function of composite home investment, let $\overline{p}_{H,t}(\Pi_t)$ be the composite price of home investment. Then the total investment expenditure can be expressed as follows:

$$\mathcal{E}_t(X_t;\Pi_t,\overline{Y}_{c,t})=\begin{cases}\overline{p}_{H,t}(\Pi_t)X_{H,t}(X_t,\overline{Y}_{c,t}), & \text{for } X_t<\overline{Y}_{c,t}/\underline{Y}_{c,t}(\Pi_t),\\ \overline{p}_t(\Pi_t)X_t-P_{c,t}\overline{Y}_{c,t}, & \text{for } X_t\geq\overline{Y}_{c,t}/\underline{Y}_{c,t}(\Pi_t),\end{cases}$$

where $\underline{Y}_{c,t}(\Pi_t)$ is the cost-minimizing ratio $Y_{c,t}/X_t$ in the absence of free child care (i.e., $\overline{Y}_{c,t}=0$) that is defined in Appendix A.4. For high levels of total investment, households invest in child care beyond the free amount and thus behave as if they receive a lump-sum transfer $P_{c,t}\overline{Y}_{c,t}$. At low levels of total investment, however, child care investment is held fixed at the free amount and households optimally choose other investments conditional on $\overline{Y}_{c,t}$.

With the non-linear total investment expenditure, the optimality condition for total investment, Equation (12), is modified as follows:

$$\mathcal{E}'_t(X_t;\Pi_t,\overline{Y}_{c,t})X_t=\frac{K_t}{u'(c_t)}.$$

Using this condition, we solve for X_t numerically.

We consider a policy that gives free child care only to families with non-college mothers in order to close the gap in total investment between non-college and college mothers that is observed in 2002. Results for single mothers, assuming no borrowing/saving, are reported in Table D-7.

Table D-7: Providing Free Child Care to Single Mothers to Eliminate Investment Gaps (Ages 5–12) by Parental Education

	Nested CES			Cobb-Douglas		
	Non-College Mothers		College Mothers	Non-College Mothers		College Mothers
	Baseline	Free Care		Baseline	Free care	
Free Child Care:						
Public Expenditure ($P_c \overline{Y}_c$)		103.63			111.21	
Quantity ($\overline{Y}_c$)		29.64			31.80	
Investment Quantities:						
Total (X)	11.04	14.61	14.61	10.89	13.82	13.82
Mother's Time (τ_m)	10.04	11.73	12.42	10.02	10.39	12.46
Goods (g)	11.97	13.76	18.09	12.59	13.05	19.75
Child Care (Y_c)	13.17	29.64	18.63	15.12	32.26	20.26

Notes: The table reports average weekly amounts of free child care, $\overline{Y}_c$, (and its cost) provided to single non-college mothers that would be needed to eliminate average differences in total investment, X, (over child ages 5–12) by parental education. Assumes all families are borrowing constrained. The table also reports endogenous responses in other investments, comparing them with baseline amounts for non-college and college mothers.

E Additional Econometrics Details

E.1 Estimation of $f(\cdot)$ for Two-Parent Households with Measurement Error in Wages

This section discusses estimation of $f(\cdot)$ for two-parent households when wages are measured with error. An analogous set of results to those in the text apply; however, the estimating equations are slightly more complicated due to the roles of both father's and mother's time inputs. Relative demand for child care vs. goods in two-parent families implies

$$\ln R_{Y_c,i} = Z_i'\phi_{Y,g} + \left[\frac{\epsilon_{Y,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln\left(1 + R_{f,i}e^{-\xi_{W_f\tau_f/g,i}} + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right) + (1-\epsilon_{Y,H})\ln \tilde{P}_{c,i} + \xi_{Y_c/g,i}, \tag{54}$$

where $\xi_{\tau_f W_f/g,i} \equiv \xi_{\tau_f,i} + \xi_{W_f,i} - \xi_{g,i}$ and other variables are defined in the main text.

With no measurement error in wages, time or goods inputs (i.e., $\xi_{\tau_f W_f/g,i} = \xi_{\tau_m W_m/g,i} = 0$), Equation (54) can be estimated via OLS.

Incorporating measurement error in all child investment inputs but assuming (i) wages for both parents are measured without error (i.e., $\xi_{W_m,i} = \xi_{W_f,i} = 0$) and (ii) no unobserved heterogeneity in either parent's child production ability (i.e., $\eta_{m,i} = \eta_{f,i} = 0$) yields the following:

$$\ln(R_{Y_c,i}) = Z_i'\phi_{Y,g} + \left[\frac{\epsilon_{Y,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln\left(1 + e^{\ln(\tilde{\Phi}_{fi})} + e^{\ln(\tilde{\Phi}_{m,i})}\right) + (1-\epsilon_{Y,H})\ln \tilde{P}_{c,i} + \xi_{Y_c/g,i}. \tag{55}$$

As with single mothers, the stated assumptions enable a two-step approach for estimating Equation (55), using predicted values from OLS estimation of Equation (13) for both fathers and mothers, $\widehat{\ln(R_{j,i})}$, in place of $\ln(\tilde{\Phi}_{j,i})$ for $j \in \{m, f\}$.

As with single mothers, we can account for measurement error in wages and inputs, as well as unobserved heterogeneity in maternal and paternal child productivity, by taking expectations of Equation (54) conditional on observed data:

$$\begin{aligned} &E\left[\ln R_{Y_c,i} \middle| Z_i, R_{f,i}, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] \\ &= Z_i'\phi_{Y,g} + \left[\tfrac{\epsilon_{Y,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] E\left[\ln\left(1 + R_{f,i}e^{-\xi_{W_f\tau_f/g,i}} + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right) \middle| R_{f,i}, R_{m,i}\right] + (1-\epsilon_{Y,H})\ln \tilde{P}_{c,i} - E[\xi_{g,i}|g_i^o]. \end{aligned}$$

Knowledge of measurement error distributions would allow for direct calculation of the conditional expectation terms on the right hand side. Alternatively, a second order Taylor approximation to integrate over measurement error and $\xi_{g,i} \sim N(0, \sigma_g^2)$ yields:

$$\begin{aligned} E\left[\ln R_{Y_c,i} \middle| Z_i, R_{f,i}, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] &\approx Z_i'\phi_{Y,g} + \left[\frac{\epsilon_{Y,H} - \epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right] \ln(1 + R_{f,i} + R_{m,i}) \\ &+ \sigma^2_{W_f\tau_f}\left[\tfrac{\epsilon_{Y,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\tfrac{R_{f,i}(1+R_{m,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) + \sigma^2_{W_m\tau_m}\left[\tfrac{\epsilon_{Y,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\tfrac{R_{m,i}(1+R_{f,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) \\ &+ \sigma_g^2\left[\tfrac{\epsilon_{Y,H}-\epsilon_{\tau,g}}{1-\epsilon_{\tau,g}}\right]\left(\tfrac{R_{f,i}+R_{m,i}}{2\left(1+R_{f,i}+R_{m,i}\right)^2}\right) - \sigma_g^2\left(\tfrac{\ln(g_i^o)-E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right) + (1-\epsilon_{Y,H})\ln(\tilde{P}_{c,i}), \end{aligned} \tag{56}$$

where $\sigma^2_{W_j\tau_j} \equiv Var(\xi_{W_j} + \xi_{\tau_j})$ for $j \in \{m, f\}$.[68] Based on this moment condition, GMM can be used to efficiently estimate the technology parameters $(\epsilon_{\tau,g}, \epsilon_{Y,H}, \phi_{Y,g})$ and measurement error variances $\left(\sigma^2_{W_m\tau_m/g}, \sigma^2_{W_m\tau_m/g}, \sigma^2_g\right)$. OLS can also be used; however, there may be some efficiency loss by not imposing parameter restrictions across terms.

[68] As with the case for single mothers, these time expenditure measurement error variances are only identified when $\epsilon_{Y,H} \neq \epsilon_{\tau,g}$.

E.2 Clustering Routine for Grouped Heterogeneity

For all mothers (indexed by n) in our main dataset, we estimate the wage equation:

$$\log(W_{n,t}) = \mu_{k(n)} + X_{n,t}\beta + \epsilon_{n,t}$$

where $k(n) \in \{1, 2, ..., K\}$ indicates the mother's fixed, unobserved type and $X_{n,t}$ includes education dummies, a second order polynomial in potential experience, and calendar year dummies.

Let $\mathcal{K} = \{k(1), k(2), ..., k(N)\}$ be the true type type of each mother. We estimate the collection of parameters $(\mathcal{K}, \beta, \mu)$ as:

$$\hat{\mathcal{K}}, \hat{\beta}, \hat{\mu} = \arg\min \sum_{n} \sum_{t=1}^{T_n} (\log(W_{n,t}) - X_{n,t}\beta - \mu_{k(n)})^2$$

using the iterative clustering routine described in Bonhomme and Manresa (2015), who also demonstrate that this estimator has regular asymptotics.

E.3 Estimation of $f(\cdot)$ and $\mathcal{H}(\cdot)$ using relative demand and skill measures

To maintain stability in estimation of the full production function using GMM (as $\rho \to 0$ or $\gamma \to 0$), we use the following specification:

$$f = \left[\left(\tilde{a}_{m,i,t}\tau_{m,i,t}^{\rho} + \tilde{a}_{f,i,t}\tau_{f,i,t}^{\rho} + (1 - \tilde{a}_{m,i,t} - \tilde{a}_{f,i,t}) g_{i,t}^{\rho} \right)^{\gamma/\rho} (1 - \tilde{a}_{Yc,i,t}) + \tilde{a}_{Yc,i,t} Y_{c,i,t}^{\gamma} \right]^{1/\gamma}$$

where

$$\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1 + \exp(Z_{i,t}\tilde{\phi}_m) + \exp(Z_{i,t}\tilde{\phi}_f)}, \; j \in \{m, f\}, \qquad \text{and} \qquad \tilde{a}_{Yc,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_Y)}{1 + \exp(Z_{i,t}\tilde{\phi}_Y)}.$$

E.3.1 Intratemporal Moments

We use GMM to jointly estimate all relative demand equations, interacting residuals from the relative demand equations with the appropriate instruments. We use the intratemporal conditions based on equations (8) and (9) to express the ratio of any two observed inputs x and y, given parameters ω, prices $\Pi_{i,t}$ and parental marital status, $M_{i,t} \in \{0, 1\}$. The residuals are given by:

$$\xi_{x/y,i,t} = \ln\left(\frac{x_{i,t}^{o}}{y_{i,t}^{o}}\right) - \ln\left(\Phi_{x,y,i,t}\right), \qquad x, y \in \{\tau_m, \tau_f, Y_c, g\}.$$

The moments we use are based on the following set of residuals (with zeros for unobserved values):

$$\xi_i = [\xi_{Y_c/m,i,97}, \; \xi_{Y_c/m,i,02}, \; \xi_{Y_c/g,i,02}, \; \xi_{m/g,i,02}, \; \xi_{f/g,i,02}, \xi_{Y_c/m,i,07}, \; \xi_{Y_c/g,i,07}, \; \xi_{m/g,i,07}, \; \xi_{f/g,i,07}],$$

interacting each residual $\xi_{x/y,i,t}$ with the vector of instruments $Z_{x/y,i,t}$, which include the observable characteristics determining the relevant factor shares ($a_{x,i,t}$ and $a_{y,i,t}$) along with relative prices (or instruments for relative prices depending on specification). In addition to the relative demand moments for parental time relative to goods and for child care relative to goods for 2002 and 2007 used in Section 4.1, we also include moments for child care relative to mother's time in 1997 (goods inputs are not measured that year). The final vector of moments is:

$$g_{1,N} = \frac{1}{N} \sum_i \begin{bmatrix} \vdots \\ \xi_{x/j,i,t} \otimes Z_{x/y,i,t} \\ \vdots \end{bmatrix}, \ \forall\ \xi_{x/y,i,t} \in \xi_i. \tag{57}$$

Residual Correlation Test Any persistent unobserved heterogeneity that is not accounted for will appear as a correlation in the residual for input ratios across years. We test the null hypothesis of no correlation using:

$$T_N = \sqrt{N} \frac{\sum_i \xi_{Y,m,i,97} \xi_{Y,m,i,02}}{\sqrt{s^2_{Y,m,97} s^2_{Y,m,02}}},$$

which is asymptotically $N(0,1)$ under the null. Here, $\xi_{Y,m,i,t}$ is the residual in the demand for childcare relative to mother's time for child i at time t and $s^2_{Y,m,t}$ is the corresponding sample variance across individuals.

E.3.2 Intertemporal Moments

To derive the moments for identifying production parameters, we start with equation (19):

$$\tilde{\Psi}_{t+5} = \sum_{s=0}^{4} \delta_2^{4-s} Z_{t+s} \hat{\phi}_\theta + \delta_1 \sum_{t=s}^{4} \delta_2^{4-s} \left[\ln \left(\frac{\bar{p}_t \tau_{m,t}}{\bar{p}_{t+s} \Phi_{m,X}(\Pi_t)} \right) + \kappa \ln \left(\frac{W_{m,t+s} + W_{f,t+s} + y_{t+s}}{W_{m,t} + W_{f,t} + y_t} \right) \right] + \delta_2^5 \tilde{\Psi}_t + \tilde{\xi}_{\theta,t+5}$$

where we omit i subscripts to save on notation. The only time-varying Z_{t+s} affecting θ_{t+s} in our empirical analysis is child's age, allowing us to write the entire first term as a function of Z_t. In the case of no borrowing/saving, the first term depends on additional structural parameters $(\alpha, \beta, r, \psi_m, \psi_f)$; however, age-specific intercept terms can absorb all of these expressions. We use a linear term in age as a first-order approximation.

With the PSID-CDS, we address measurement error in child human capital $(\Psi_{i,t})$ and mother's time using two age-normalized measures of cognitive ability from the *Letter-Word* $(LW_{i,t})$ and *Applied Problems* $(AP_{i,t})$ modules of the Woodcock-Johnson aptitude test. We write these as:

$$S_{i,t} = \lambda_S \tilde{\Psi}_{i,t} + \xi_{S,i,t}, \quad S \in \{LW, AP\},\ t \in \{1997, 2002, 2007\}.$$

These measurement assumptions require a normalization on the factor loading for one measure, as in Cunha, Heckman, and Schennach (2010). We set $\lambda_{LW} = 1$, leaving the factor loading on the Applied Problems score (λ_{AP}) to be estimated. Substituting these noisy measures into the outcome equation above gives:

$$\lambda_S^{-1} S_{t+5} = Z_t \overline{\phi}_\theta + \delta_1 \sum_{s=0}^{4} \delta_2^{4-s} \left[\ln \left(\frac{\bar{p}_t \tau^o_{m,t}}{\bar{p}_{t+s} \Phi_{m,X}(\Pi_t)} \right) + \kappa \ln \left(\frac{W_{m,t+s} + W_{f,t+s} + y_{t+s}}{W_{m,t} + W_{f,t} + y_t} \right) \right] + \delta_2^5 LW_t + \tilde{\xi}_{\Psi,S,t}$$

for $S \in \{AP, LW\}$, where $Z_t \overline{\phi}_\theta$ reflects our approximation for the first term in Equation (19) and $\tilde{\xi}_{\Psi,S,t}$ collects the measurement error terms $\xi_{m,t}$ and $\xi_{AP,t}$, and the innovation term $\tilde{\xi}_{\theta,t+5}$.

Our second set of moments for production parameters are now given by:

$$g_{2,N} = \frac{1}{N}\sum_{n}\begin{bmatrix} \vdots \\ \tilde{\xi}_{\Psi,S,i,t} \otimes Z_{\Psi,i,t} \\ \vdots \end{bmatrix} \; t \in \{1997, 2002\}, S \in \{AP, LW\}, \tag{58}$$

where the instrument set $Z_{\Psi,i,t}$ contains all $Z_{i,t}$ that are permitted to influence $\theta_{i,t}$, along with $\ln(\tau^{o}_{m,i,t+5})$ to instrument for $\ln(\tau^{o}_{m,i,t})$.

In order to identify the factor loading λ_{AP}, we use the assumption that measurement error is independent over time to write:

$$\lambda_{AP} = \frac{Cov(AP_{i,t+5}, LW_{i,t})}{Cov(LW_{i,t+5}, LW_{i,t})}, \qquad \lambda^2_{AP} = \frac{Cov(AP_{i,t+5}, AP_{i,t})}{Cov(LW_{i,t+5}, LW_{i,t})}.$$

Because we normalize our measurements to have mean zero, these two identifying conditions can be written as the following pair of moments:

$$E\left[(AP_{i,t+5} - \lambda_{AP} LW_{i,t+5}) LW_{i,t}\right] = 0 \qquad \text{and} \qquad E\left[AP_{i,t+5} AP_{i,0} - \lambda^2_{AP} LW_{i,t+5} LW_{i,t}\right] = 0. \tag{59}$$

The full estimation procedure conducts optimally weighted GMM by stacking the moment conditions on input ratios, the moment conditions on the achievement equation, and the moment conditions derived from our measurement assumptions above. The parameters to be estimated are $\omega = (\rho, \gamma, \tilde{\phi}_m, \tilde{\phi}_f, \tilde{\phi}_Y)$, $\delta = (\delta_1, \delta_2)$, $\overline{\phi}_\theta$, and λ_{AP}.

E.3.3 Relaxing and Testing Relative Demand and Production Parameters

Recall that relative input demand is determined by the set of parameters $\omega = (\rho, \gamma, \phi_m, \phi_f, \phi_Y)$. Let $\tilde{\omega}$ indicate the parameter values that are perceived by parents in that they determine relative demand but do not necessarily enter the production function (i.e., ω need not equal $\tilde{\omega}$).

Let each relative input ratio $\Phi_{x,y,i,t}$ now depend on prices, marital status, and $\tilde{\omega}$, resulting in a moment condition for relative demand that depends only only on $\tilde{\omega}$: $g_{1,N}(\tilde{\omega})$.

Next, observe that:

$$X_{n,t} = \underbrace{\left(\left(a_{m,i,t} + a_{g,i,t}\Phi^{\rho}_{g,m,i,t} + a_{f,i,t}\Phi^{\rho}_{f,m,i,t}\right)^{\gamma/\rho}(1 - a_{Y,i,t}) + a_{Y,i,t}\Phi^{\gamma}_{Y,m,i,t}\right)^{1/\gamma}}_{\equiv \Phi_{X,m,i,t}} \tau_{m,i,t},$$

which yields a general expression for $\Phi_{X,m,i,t}$ that depends on the perceived technology parameters, $\tilde{\omega}$, through $(\Phi_{g,m,i,t}, \Phi_{g,m,i,t}, \Phi_{Y,m,i,t})$, as well as the true technology parameters, ω.

Let $\overline{p}(\Pi_{n,t}, Z_{n,t}, \omega, \tilde{\omega})$ indicate the *effective* composite price of X, which reflects the dollars of expenditure per investment unit as defined in Equation (6). This price now depends on both perceived and true technology parameters:

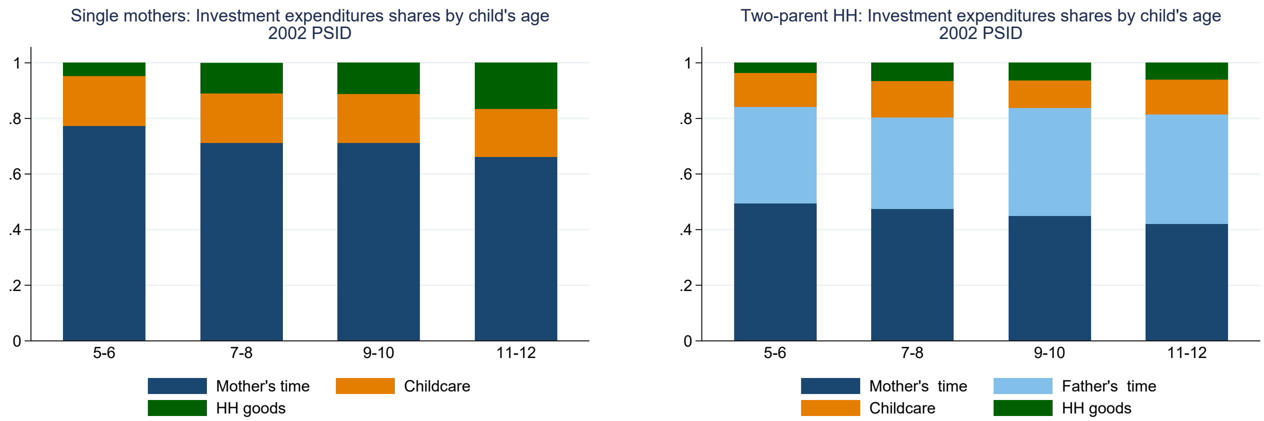
$$\bar{p}(\Pi_{i,t}, Z_{i,t}, \omega, \tilde{\omega}) = \frac{p_{i,t} + P_{Y,i,t}\Phi_{Y,i,t} + W_{m,i,t}\Phi_{m,i,t} + W_{f,i,t}\Phi_{f,i,t}}{\left[\left(a_{m,i,t}\Phi^{\rho}_{m,i,t} + a_{f,i,t}\Phi^{\rho}_{f,i,t} + 1 - a_{m,i,t} - a_{f,i,t}\right)^{\gamma/\rho}(1 - a_{Y,i,t}) + a_{Y,i,t}\Phi^{\gamma}_{Y,i,t}\right]^{1/\gamma}}$$

Combining moment conditions now gives:

$$g_N(\delta, \hat{\phi}_\theta, \omega, \tilde{\omega}) = \begin{bmatrix} g_{1,N}(\tilde{\omega}) \\ g_{2,N}(\delta, \overline{\phi}_\theta, \lambda_{AP}, \omega, \tilde{\omega}) \end{bmatrix}.$$

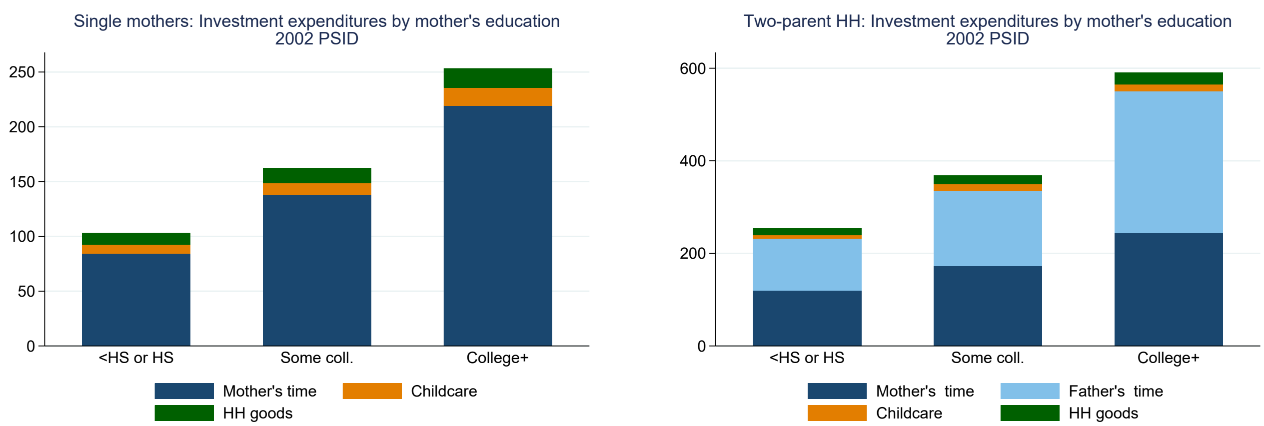
From here, we can directly apply three tests of the null hypothesis that $\tilde{\omega} = \omega$ as described in Section 9.2 of Newey and McFadden (1994). The Lagrange Multiplier test statistic is an appropriately normalized derivative of the GMM criterion with respect to the retricted parameters at the restricted estimates ($\omega = \tilde{\omega}$). The Wald statistic takes a weighted average of the squared distance between $\hat{\omega}$ and $\hat{\tilde{\omega}}$ when they minimize the GMM criterion in an unconstrained way, and the Distance Metric compares the value of the GMM criterion itself at the constrained vs unconstrained estimates. Each statistic follows a Chi-squared distribution with degrees of freedom equal to the number of constraints. In our analysis, we estimate the constrained model and test individual parameter restrictions using the Lagrange Multiplier statistic. We re-estimate the model by relacing all parameters that fail this test at 5% significance. At those estimates we can conduct the distance metric and Wald tests.

F Additional Empirical Results



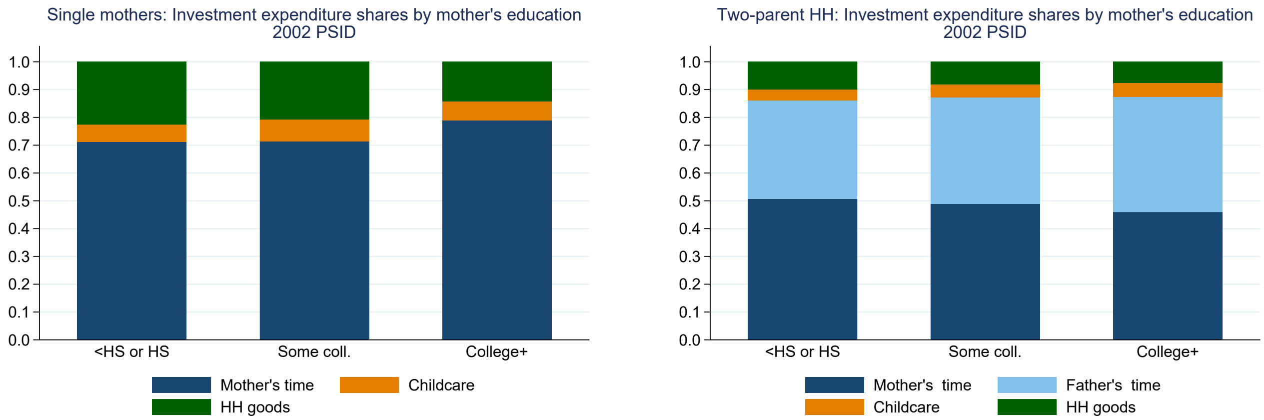
(a) Single mothers (b) Two-parent households

Figure F-1: Expenditure shares by child's age (PSID, 2002)



(a) Single mothers (b) Two-parent households

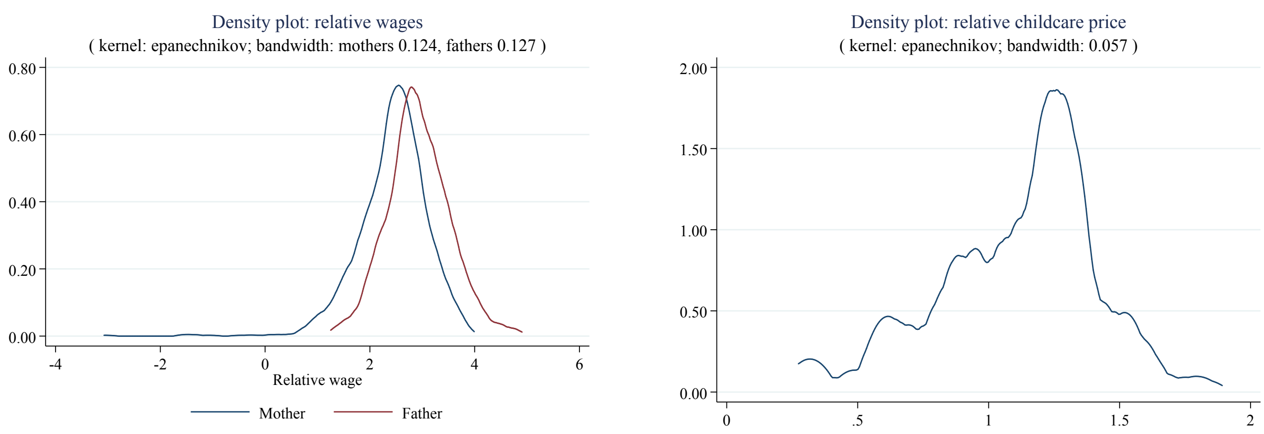
Figure F-2: Weekly child investment expenditures by mother's education, includes families with zero child care spending (PSID, 2002)



(a) Single mothers

(b) Two-parent households

Figure F-3: Expenditure shares by mother's education, includes families with zero child care spending (PSID, 2002)



(a) $\ln(\tilde{W}_m)$ and $\ln(\tilde{W}_f)$

(b) $\ln(\tilde{P}_c)$

Figure F-4: Distributions of log relative input prices (PSID, 2002 and 2007)

Table F-1: Summary statistics for restricted samples: 2002 and 2007

	(1)	(2)	(3)	(4)	(5)	(6)
	High pred. prob. of work (mothers)			Positive child care spending		
	Sample Size	Mean	SD	Sample Size	Mean	SD
$\ln(\tilde{W}_m)$	928	2.49	0.62	384	2.56	0.52
$\ln(\tilde{W}_f)$	662	2.98	0.60	247	2.86	0.53
$\ln(\tilde{P}_c)$	1156	1.10	0.32	423	1.10	0.33
Child's age	1156	9.60	2.05	423	8.34	1.95
Mother HS grad	1156	0.31	0.46	422	0.25	0.43
Mother some coll.	1156	0.35	0.48	422	0.37	0.48
Mother coll+	1156	0.33	0.47	422	0.33	0.47
Mother's age	1156	37.87	6.36	423	35.92	6.22
Father HS grad	744	0.40	0.49	265	0.35	0.48
Father some coll.	744	0.26	0.44	265	0.23	0.42
Father coll+	744	0.32	0.47	265	0.34	0.48
Father's age	744	40.72	6.99	264	38.71	6.73
Mother white	1156	0.57	0.50	421	0.56	0.50
Num children age 0-5	1156	0.14	0.37	423	0.34	0.50
Num of children	1156	1.98	0.70	423	1.87	0.61
Year = 2007	1156	0.20	0.40	423	0.04	0.20

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Columns 1-3 based on sample of mothers with predicted probability of work of at least 0.75. See Table F-3 for model of predicted probability of work. Columns 4-6 based on sample of mothers with positive childcare spending.

Table F-2: Log wage regressions for parents

	(1) Single Mothers	(2) Married Mothers	(3) Married Fathers
HS graduate			0.307*
			(0.066)
Some college	0.246*	0.257*	0.476*
	(0.053)	(0.049)	(0.069)
College +	0.526*	0.585*	0.781*
	(0.067)	(0.048)	(0.066)
Age	0.068	0.055	0.072*
	(0.038)	(0.033)	(0.020)
Age-squared	-0.001	-0.001	-0.001*
	(0.001)	(0.000)	(0.000)
Mother white	0.133*	-0.044	0.178*
	(0.050)	(0.042)	(0.041)
Constant	0.614	0.963	0.769
	(0.679)	(0.620)	(0.401)
R-squared	0.149	0.179	0.223
Sample size	542	932	1182

Notes: Sample includes families in 1997, 2002, or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) included. See Table F-3 for model of predicted probability of work. * significant at 0.05 level.

Table F-3: Predicted probability (average derivatives) of work probits for parents

	(1) Single Mothers	(2) Married Mothers	(3) Married Fathers
Mother HS grad	0.115*	0.075	-0.024
	(0.038)	(0.045)	(0.026)
Mother some coll.	0.153*	0.111*	-0.008
	(0.039)	(0.047)	(0.027)
Mother coll+	0.271*	0.189*	0.028
	(0.053)	(0.049)	(0.031)
Mother's age	-0.005*	-0.000	0.001
	(0.002)	(0.003)	(0.002)
Mother white	0.047	-0.019	0.056*
	(0.029)	(0.022)	(0.012)
Num children age 0-5	-0.024	-0.034	0.006
	(0.055)	(0.038)	(0.023)
Num of children	-0.004	-0.001	-0.010
	(0.017)	(0.014)	(0.007)
age of youngest child	0.007	0.012	0.002
	(0.009)	(0.007)	(0.004)
year = 2002	0.018	0.074*	0.071*
	(0.028)	(0.021)	(0.012)
year = 2007	0.005	-0.016	0.075*
	(0.047)	(0.040)	(0.017)
Father HS grad		0.123*	0.007
		(0.036)	(0.020)
Father some coll.		0.110*	0.005
		(0.041)	(0.022)
Father coll+		0.023	0.071*
		(0.041)	(0.026)
Father's age		-0.002	-0.002
		(0.002)	(0.001)
Sample size	824	1,753	1,737

Notes: Sample includes families in 1997, 2002, or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. All specifications include CDS child age dummies. * significant at 0.05 level.

Table F-4: First-stage estimates for mother time/goods relative demand using different predicted wage measures as instruments

	(1) Uses All Variation	(2) Excludes Avg. State Diff.	(3) Excludes Avg. Occ. Diff.	(4) Excludes Avg. State & Occ. Diff.
Mother's predicted log wage	1.084*	1.002*	0.762*	0.373
	(0.103)	(0.110)	(0.178)	(0.209)
Married	-0.045	-0.057	-0.055	-0.058
	(0.046)	(0.047)	(0.049)	(0.049)
Child's age	-0.009	-0.011	-0.012	-0.013
	(0.011)	(0.011)	(0.011)	(0.012)
Mother some coll.	0.191*	0.201*	0.270*	0.271*
	(0.049)	(0.050)	(0.051)	(0.052)
Mother coll+	0.433*	0.448*	0.610*	0.596*
	(0.052)	(0.053)	(0.054)	(0.055)
Mother's age	0.009*	0.012*	0.009*	0.011*
	(0.004)	(0.004)	(0.004)	(0.004)
Mother white	0.026	0.089*	0.073	0.096*
	(0.043)	(0.044)	(0.046)	(0.048)
Num children ages 0-5	0.039	0.032	0.042	0.035
	(0.060)	(0.062)	(0.064)	(0.065)
Num of children	-0.047	-0.055	-0.069*	-0.074*
	(0.030)	(0.030)	(0.031)	(0.032)
Constant	0.295	0.428	0.537	1.323*
	(0.229)	(0.241)	(0.395)	(0.453)
F-statistic Excluded Instrument	111.60	82.83	18.36	3.19
Sample size	720	720	720	720

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table F-3 for model of predicted probability of work. This table reports effects of predicted log wage instruments and other exogenous family characteristics on log relative wages, $\ln(\tilde{W}_{m,t})$, for mothers. Using the 2000 Census, predicted log wages are obtained from gender-specific regressions of log wages on an indicator for race (white/non-white), potential experience and experience-squared, educational attainment (<12 years, 12 years, 13–15 years, 16 years, 17+ years), 16 industry dummies, 97 occupation dummies (minor 2000 SOC codes), state dummies, interactions of race and education dummies with experience, and interactions of race and occupation dummies with state dummies. Column (1) uses predicted log wages. Column (2) eliminates average differences across states from predicted log wages, column (3) eliminates average differences across occupations, and column (4) eliminates average differences across states and occupations. * significant at 0.05 level.

Table F-5: 2SLS estimates for mother time/goods relative demand using different predicted wage measures as instruments

	(1) Uses All Variation	(2) Excludes Avg. State Diff.	(3) Excludes Avg. Occ. Diff.	(4) Excludes Avg. State & Occ. Diff.
$\ln(\tilde{W}_{m,t})$	0.553*	0.359	0.799	-0.450
	(0.196)	(0.226)	(0.457)	(1.243)
Married	-0.071	-0.081	-0.058	-0.123
	(0.096)	(0.097)	(0.098)	(0.127)
Child's age	-0.140*	-0.143*	-0.137*	-0.154*
	(0.022)	(0.023)	(0.023)	(0.031)
Mother some coll.	0.026	0.078	-0.040	0.296
	(0.113)	(0.118)	(0.158)	(0.353)
Mother coll+	-0.119	-0.007	-0.262	0.463
	(0.155)	(0.168)	(0.285)	(0.731)
Mother's age	-0.007	-0.005	-0.010	0.004
	(0.008)	(0.008)	(0.009)	(0.016)
Mother white	-0.233*	-0.218*	-0.251*	-0.158
	(0.091)	(0.092)	(0.096)	(0.138)
Num children ages 0-5	0.168	0.174	0.159	0.201
	(0.126)	(0.127)	(0.127)	(0.150)
Num of children	0.082	0.068	0.101	0.008
	(0.063)	(0.064)	(0.070)	(0.116)
Constant	2.398*	2.800*	1.887	4.479
	(0.520)	(0.572)	(1.002)	(2.606)
Sample size	720	720	720	720

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5–12 and only 1–2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table F-3 for model of predicted probability of work. Using the 2000 Census, predicted log wages, used as instruments for $\ln(\tilde{W}_{m,t})$, are obtained from gender-specific regressions of log wages on an indicator for race (white/non-white), potential experience and experience-squared, educational attainment (<12 years, 12 years, 13–15 years, 16 years, 17+ years), 16 industry dummies, 97 occupation dummies (minor 2000 SOC codes), state dummies, interactions of race and education dummies with experience, and interactions of race and occupation dummies with state dummies. Column (1) uses predicted log wages as instruments. Column (2) eliminates average differences across states from predicted log wages, column (3) eliminates average differences across occupations, and column (4) eliminates average differences across states and occupations. * significant at 0.05 level.

Table F-6: OLS & 2SLS estimates for mother time/goods relative demand for different selection on predicted probability of work

	(1)	(2)	(3)	(4)	(5)	(6)
	OLS			2SLS (instrument: predicted log wage)		
	Base	All	$Pr(work) \geq 0.85$	Base	All	$Pr(work) \geq 0.85$
$\ln(\tilde{W}_{m,t})$	0.646*	0.662*	0.624*	0.749*	0.413*	0.531*
	(0.071)	(0.065)	(0.101)	(0.216)	(0.185)	(0.235)
Married	-0.074	-0.079	-0.146	-0.069	-0.077	-0.141
	(0.095)	(0.089)	(0.116)	(0.095)	(0.091)	(0.116)
Child's age	-0.141*	-0.131*	-0.146*	-0.139*	-0.133*	-0.146*
	(0.022)	(0.020)	(0.031)	(0.022)	(0.020)	(0.031)
Mother some coll.	0.011	-0.071	0.021	-0.018	-0.003	0.019
	(0.102)	(0.092)	(0.145)	(0.117)	(0.107)	(0.146)
Mother coll+	-0.157	-0.226*	-0.152	-0.218	-0.088	-0.145
	(0.112)	(0.103)	(0.153)	(0.164)	(0.149)	(0.172)
Mother's age	-0.008	-0.006	-0.005	-0.009	-0.004	-0.003
	(0.008)	(0.007)	(0.010)	(0.008)	(0.007)	(0.011)
Mother white	-0.243*	-0.152	-0.283*	-0.249*	-0.128	-0.281*
	(0.089)	(0.084)	(0.111)	(0.090)	(0.086)	(0.111)
Num. children	0.158	0.147	-0.024	0.155	0.164	-0.007
ages 0-5	(0.125)	(0.104)	(0.215)	(0.125)	(0.106)	(0.217)
Num. of children	0.089	0.100	0.106	0.097	0.080	0.091
	(0.062)	(0.056)	(0.080)	(0.063)	(0.058)	(0.081)
Constant	2.213*	1.992*	2.224*	1.999*	2.501*	2.425*
	(0.355)	(0.327)	(0.461)	(0.553)	(0.488)	(0.627)
R-squared	0.190	0.179	0.163	0.187	0.161	0.151
Sample size	727	860	417	727	851	412

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. See Table F-3 for model of predicted probability of work. * significant at 0.05 level.

Table F-7: 2SLS estimates for parental time vs. goods relative demand

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.553*	0.281	0.697*	0.346
	(0.196)	(0.387)	(0.228)	(0.257)
Married	-0.071			
	(0.096)			
Child's age	-0.140*	-0.176*	-0.129*	-0.099*
	(0.022)	(0.045)	(0.026)	(0.027)
Parent some coll.	0.026	0.286	-0.156	-0.020
	(0.113)	(0.189)	(0.142)	(0.154)
Parent coll+	-0.119	0.181	-0.320	0.276
	(0.155)	(0.279)	(0.188)	(0.183)
Parent's age	-0.007	-0.009	-0.007	-0.010
	(0.008)	(0.014)	(0.009)	(0.009)
Mother white	-0.233*	-0.365*	-0.170	0.001
	(0.091)	(0.175)	(0.107)	(0.128)
Num children age 0-5	0.168	-0.121	0.292*	0.154
	(0.126)	(0.240)	(0.147)	(0.135)
Num of children	0.082	0.028	0.110	0.182*
	(0.063)	(0.117)	(0.076)	(0.081)
Constant	2.398*	3.502*	1.858*	2.028*
	(0.520)	(1.086)	(0.582)	(0.683)
R-squared	0.181	0.155	0.195	0.136
Sample size	720	233	487	578

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) are included. See Table F-3 for model of predicted probability of work. Specification for mothers (fathers) includes mother's (father's) relative wage, education indicators, and age. All columns instrument for $\ln(\tilde{W}_{j,i})$ using predicted log wages from 2000 Census as instruments (see text for details). * significant at 0.05 level.

Table F-8: OLS estimates for parental time vs. goods relative demand including parental log wage fixed effects, by parent type

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.758*	0.767*	0.790*	0.779*
	(0.092)	(0.198)	(0.106)	(0.121)
Married	0.022			
	(0.108)			
Child's age	-0.147*	-0.163*	-0.144*	-0.127*
	(0.024)	(0.053)	(0.027)	(0.030)
Parent's log wage FE	-0.346*	-0.089	-0.503*	-0.171
	(0.114)	(0.198)	(0.141)	(0.122)
Mother white	-0.328*	-0.579*	-0.217	-0.287
	(0.102)	(0.192)	(0.121)	(0.155)
Num children age 0-5	0.163	-0.011	0.222	0.303
	(0.169)	(0.360)	(0.190)	(0.187)
Num of children	0.027	-0.011	0.046	0.169
	(0.066)	(0.117)	(0.082)	(0.092)
Constant	1.745*	2.055*	1.542*	0.959*
	(0.366)	(0.829)	(0.403)	(0.482)
R-squared	0.193	0.215	0.197	0.171
Sample size	562	162	400	413

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers (fathers) with predicted probability of work of at least 0.75 (0.9) are included. See Table F-3 for model of predicted probability of work. Specification for mothers (fathers) includes mother's (father's) relative wage, and mother's (father's) log wage fixed effects. * significant at 0.05 level.

Table F-9: Heckman two-step estimates for mother time/goods relative demand (two-parent households)

	(1) father only	(2) father only	(3) P_c only	(4) Both	(5) Both, P(work) $\geq$ .75
			A. Relative demand		
$\ln(\tilde{W}_{m,t})$	0.644*	0.640*	0.640*	0.640*	0.633*
	(0.075)	(0.075)	(0.128)	(0.075)	(0.079)
Child's age	-0.134*	-0.133*	-0.171	-0.134*	-0.142*
	(0.028)	(0.027)	(0.112)	(0.027)	(0.030)
Mother HS grad	-0.110				
	(0.248)				
Mother some coll.	-0.335	-0.230	-0.559	-0.237	-0.148
	(0.272)	(0.127)	(0.791)	(0.126)	(0.136)
Mother coll+	-0.472	-0.361*	-0.846	-0.371*	-0.302*
	(0.296)	(0.147)	(1.187)	(0.146)	(0.143)
Mother's age	-0.001	-0.002	0.016	-0.001	-0.004
	(0.009)	(0.009)	(0.048)	(0.009)	(0.010)
Mother white	-0.073	-0.084	0.171	-0.079	-0.154
	(0.111)	(0.108)	(0.651)	(0.107)	(0.112)
Num children age 0-5	0.316*	0.311*	0.469	0.315*	0.283
	(0.126)	(0.125)	(0.467)	(0.125)	(0.149)
Num of children	0.105	0.107	0.176	0.108	0.116
	(0.070)	(0.069)	(0.213)	(0.069)	(0.077)
Year = 2007	0.090	0.086	0.336	0.092	0.084
	(0.133)	(0.133)	(0.606)	(0.133)	(0.139)
Constant	1.952*	1.853*	2.287	1.865*	2.036*
	(0.469)	(0.409)	(1.409)	(0.410)	(0.435)
			B. Positive hours worked by mother		
Child's age	0.037	0.038	0.039	0.039	0.053
	(0.032)	(0.032)	(0.031)	(0.032)	(0.038)
Mother HS grad	0.178				
	(0.234)				
Mother some coll.	0.503*	0.347*	0.287*	0.343*	0.424*
	(0.248)	(0.140)	(0.126)	(0.141)	(0.168)
Mother coll+	0.781*	0.625*	0.453*	0.619*	0.669*
	(0.256)	(0.153)	(0.126)	(0.153)	(0.185)
Mother's age	-0.014	-0.013	-0.019	-0.015	-0.018
	(0.015)	(0.015)	(0.010)	(0.015)	(0.018)
Mother white	-0.185	-0.173	-0.268*	-0.192	-0.157
	(0.128)	(0.127)	(0.125)	(0.128)	(0.153)
Num children age 0-5	-0.118	-0.112	-0.154	-0.116	0.170
	(0.139)	(0.139)	(0.135)	(0.139)	(0.212)
Num of children	-0.041	-0.048	-0.061	-0.047	-0.073
	(0.079)	(0.079)	(0.077)	(0.079)	(0.091)
Year = 2007	-0.266	-0.275	-0.219	-0.288*	-0.264
	(0.144)	(0.143)	(0.140)	(0.144)	(0.175)
Father HS grad	0.433*	0.478*		0.494*	0.616
	(0.191)	(0.182)		(0.183)	(0.388)
Father some coll.	0.228	0.269		0.276	0.338
	(0.209)	(0.202)		(0.202)	(0.397)
Father coll+	-0.174	-0.137		-0.135	-0.198
	(0.216)	(0.210)		(0.211)	(0.411)
Father's age	0.007	0.007		0.007	0.013
	(0.013)	(0.013)		(0.013)	(0.015)
$\ln(\tilde{P}_{c,t})$			0.082	0.161	0.116
			(0.158)	(0.162)	(0.190)
Constant	0.349	0.460	1.168*	0.330	0.043
	(0.512)	(0.491)	(0.458)	(0.509)	(0.660)
Inverse Mill's Ratio	-0.196	-0.131	-2.430	-0.186	-0.183
	(0.455)	(0.428)	(5.697)	(0.420)	(0.427)
Num. Pos. Hours	582	582	593	582	491
Sample size	756	756	771	756	610

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table F-10: OLS estimates for mother time/goods relative demand by child age

	(1) All	(2) Ages 5-8	(3) Ages 9-12
$\ln(\tilde{W}_{m,t})$	0.646*	0.648*	0.659*
	(0.071)	(0.135)	(0.086)
Married	-0.074	-0.114	-0.036
	(0.095)	(0.158)	(0.120)
Child's age	-0.141*	-0.243*	-0.160*
	(0.022)	(0.079)	(0.048)
Mother some coll.	0.011	0.263	-0.101
	(0.102)	(0.174)	(0.127)
Mother coll+	-0.157	-0.109	-0.180
	(0.112)	(0.192)	(0.139)
Mother's age	-0.008	-0.009	-0.008
	(0.008)	(0.012)	(0.010)
Mother white	-0.243*	-0.125	-0.323*
	(0.089)	(0.144)	(0.115)
Num children ages 0-5	0.158	0.130	0.095
	(0.125)	(0.162)	(0.207)
Num of children	0.089	0.062	0.087
	(0.062)	(0.120)	(0.073)
Constant	2.213*	2.838*	2.491*
	(0.355)	(0.744)	(0.639)
R-squared	0.190	0.170	0.151
Residual sum of squares	826.887	213.167	606.904
F-test equality by child's age (p-value)		0.825	
Sample size	727	224	503

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table F-3 for model of predicted probability of work. * significant at 0.05 level.

Table F-11: OLS estimates for mother time/goods relative demand by father's wage

	(1) All	(2) Below Median	(3) Above Median
$\ln(\tilde{W}_{m,t})$	0.716*	0.797*	0.647*
	(0.082)	(0.111)	(0.124)
Child's age	-0.124*	-0.099*	-0.152*
	(0.027)	(0.037)	(0.039)
Mother some coll.	-0.145	-0.293	0.087
	(0.133)	(0.167)	(0.221)
Mother coll+	-0.298*	-0.312	-0.224
	(0.136)	(0.182)	(0.213)
Mother's age	-0.007	-0.008	0.002
	(0.010)	(0.013)	(0.016)
Mother white	-0.119	-0.141	-0.031
	(0.112)	(0.140)	(0.194)
Num children ages 0-5	0.315*	0.373*	0.272
	(0.149)	(0.189)	(0.241)
Num of children	0.084	0.072	0.115
	(0.080)	(0.108)	(0.121)
Constant	1.779*	1.525*	1.576*
	(0.443)	(0.599)	(0.763)
R-squared	0.222	0.258	0.203
Residual sum of squares	457.346	217.931	233.663
F-test equality by father's wage (p-value)		0.786	
Sample size	451	231	220

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table F-3 for model of predicted probability of work. * significant at 0.05 level.

Table F-12: OLS estimates for mother time/goods relative demand conditioning on 1997 AP and LW scores

	(1) Baseline	(2) Includes 1997 Achievement	(3) Below Median 1997 Achieve.	(4) Above Median 1997 Achieve.
$\ln(\tilde{W}_{m,t})$	0.557*	0.562*	0.327	0.676*
	(0.117)	(0.118)	(0.195)	(0.142)
Married	-0.118	-0.119	-0.145	-0.054
	(0.147)	(0.148)	(0.210)	(0.211)
Child's age	-0.109*	-0.106	-0.084	-0.121
	(0.054)	(0.054)	(0.083)	(0.070)
Mother some coll.	-0.108	-0.101	-0.303	0.090
	(0.155)	(0.156)	(0.215)	(0.236)
Mother coll+	-0.243	-0.227	-0.113	-0.210
	(0.172)	(0.176)	(0.267)	(0.237)
Mother's age	0.001	0.002	0.018	-0.020
	(0.012)	(0.012)	(0.017)	(0.016)
Mother white	-0.279	-0.268	-0.543*	0.138
	(0.146)	(0.148)	(0.212)	(0.210)
Num children ages 0-5	0.069	0.081	-0.222	0.262
	(0.232)	(0.234)	(0.368)	(0.305)
Num of children	0.120	0.118	0.084	0.130
	(0.089)	(0.089)	(0.127)	(0.128)
1997 Achievement		-0.029		
		(0.068)		
Constant	1.810*	1.741*	1.783	1.983*
	(0.682)	(0.701)	(1.054)	(0.908)
R-squared	0.094	0.095	0.088	0.191
Residual sum of squares	400.478	400.251	207.122	173.339
F-test equality by 1997 Achieve. (p-value)			0.085	
Sample Size	339	339	165	174

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Only mothers with predicted probability of work of at least 0.75 included. See Table F-3 for model of predicted probability of work. * significant at 0.05 level.

Table F-13: Probit estimates for positive child care expenditures

	Avg. Derivative
$\ln(\tilde{P}_{c,t})$	0.035
	(0.035)
Married	-0.028
	(0.026)
Child's age	-0.040*
	(0.006)
Mother some coll.	0.098*
	(0.025)
Mother coll+	0.120*
	(0.028)
Mother's age	-0.002
	(0.002)
Mother white	-0.029
	(0.025)
Num children age 0-5	0.089*
	(0.027)
Num of children	-0.054*
	(0.022)
Year = 2007	-0.194*
	(0.028)
HH Head lives in same state	-0.015
	(0.024)
Any children ages 13+	-0.045
	(0.033)
Sample size	1,391

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table F-14: Heckman two-step estimates for childcare/goods relative demand

	(1)	(2)	(3)
A. Relative demand			
$\ln(\tilde{P}_{c,t})$	0.655*	0.653*	0.456*
	(0.219)	(0.217)	(0.214)
Married	0.825	0.833	0.085
	(0.626)	(0.622)	(0.635)
Child's age	-0.242*	-0.233*	-0.129
	(0.063)	(0.067)	(0.066)
Mother some coll.	0.253	0.232	0.175
	(0.203)	(0.206)	(0.187)
Mother coll+	0.054	0.028	-0.062
	(0.234)	(0.241)	(0.208)
Mother's age	-0.004	-0.004	-0.002
	(0.014)	(0.014)	(0.014)
Marr x Father some coll.	0.071	0.077	0.172
	(0.223)	(0.221)	(0.217)
Marr x Father coll+	-0.455	-0.453	-0.660*
	(0.236)	(0.233)	(0.244)
Marr x Father's age	-0.018	-0.018	-0.006
	(0.016)	(0.016)	(0.016)
Mother white	-0.342*	-0.345*	-0.214
	(0.152)	(0.151)	(0.150)
Num children age 0-5	0.175	0.158	0.126
	(0.177)	(0.179)	(0.184)
Num of children	-0.078	-0.063	0.010
	(0.144)	(0.148)	(0.153)
$\ln(1 + e^{\Phi_{m,t}} + Marr \cdot e^{\Phi_{f,t}})$			0.544
			(0.306)
Constant	1.228	1.206	-0.491
	(0.643)	(0.638)	(1.146)
B. Positive child care expenditure			
$\ln(\tilde{P}_{c,t})$	0.103	0.102	-0.084
	(0.135)	(0.134)	(0.158)
Married	0.144	0.285	-0.191
	(0.379)	(0.375)	(0.469)
Child's age	-0.135*	-0.134*	-0.120*
	(0.023)	(0.023)	(0.034)
Mother some coll.	0.339*	0.351*	0.282*
	(0.101)	(0.101)	(0.118)
Mother coll+	0.501*	0.508*	0.398*
	(0.114)	(0.113)	(0.133)
Mother's age	-0.009	-0.004	0.001
	(0.009)	(0.009)	(0.011)
Marr x Father some coll.	-0.060	-0.067	-0.105

Table F-14 continued from previous page

	(0.135)	(0.135)	(0.154)
Marr x Father coll+	-0.009	-0.009	-0.100
	(0.146)	(0.146)	(0.178)
Marr x Father's age	-0.006	-0.009	-0.001
	(0.009)	(0.009)	(0.011)
Mother white	-0.082	-0.067	0.026
	(0.095)	(0.094)	(0.111)
Num children age 0-5	0.298*	0.301*	0.339*
	(0.106)	(0.105)	(0.129)
Num of children	-0.243*	-0.233*	-0.244*
	(0.094)	(0.094)	(0.111)
Year = 2007	-0.757*	-0.751*	-0.772*
	(0.142)	(0.142)	(0.172)
Household head live in birth state	-0.042	-0.030	-0.149
	(0.092)	(0.092)	(0.107)
Live w/older relative	-0.540*		
	(0.225)		
Any children ages 13+	-0.119	-0.128	-0.313*
	(0.124)	(0.124)	(0.145)
2+ children ages 13+	0.141	0.129	0.116
	(0.217)	(0.217)	(0.250)
$\ln(1 + e^{\Phi_{m,t}} + Marr \cdot e^{\Phi_{f,t}})$			0.322
			(0.186)
Constant	1.286*	1.043*	0.547
	(0.452)	(0.440)	(0.774)
Inverse Mill's ratio	0.948*	0.874	0.467
	(0.421)	(0.460)	(0.426)
Num. pos. child care exp.	338	338	302
Sample size	1318	1318	930

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table F-15: OLS estimates for child care/goods relative demand

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Single mothers				Two-parent households		
$\ln(\tilde{P}_{c,t})$	0.853*	0.656*	0.645*	0.737*	0.589*	0.428	0.464	0.609*
	(0.309)	(0.303)	(0.311)	(0.287)	(0.275)	(0.315)	(0.293)	(0.265)
Child's age	-0.132*	-0.048	-0.078	-0.047	-0.119*	-0.112	-0.096	-0.065
	(0.057)	(0.059)	(0.085)	(0.056)	(0.046)	(0.057)	(0.066)	(0.048)
Mother some coll.	0.074	-0.010	0.018	0.260	-0.191	0.076	-0.062	0.146
	(0.260)	(0.261)	(0.289)	(0.259)	(0.215)	(0.252)	(0.227)	(0.214)
Mother coll+	-0.091	-0.196	-0.200	0.291	-0.496*	-0.038	-0.291	0.422
	(0.283)	(0.277)	(0.317)	(0.296)	(0.219)	(0.252)	(0.232)	(0.222)
Mother's age	-0.005	0.006	-0.002	0.016	0.007	-0.024	0.002	-0.014
	(0.017)	(0.017)	(0.017)	(0.016)	(0.024)	(0.030)	(0.026)	(0.025)
Mother white	-0.789*	-0.706*	-0.692*	-0.429	-0.101	-0.107	-0.015	0.086
	(0.240)	(0.229)	(0.260)	(0.229)	(0.177)	(0.198)	(0.189)	(0.168)
Num children age 0-5	-0.289	0.055	-0.164	0.205	0.021	0.200	0.081	0.357
	(0.268)	(0.258)	(0.274)	(0.254)	(0.175)	(0.213)	(0.216)	(0.181)
Num of children	0.056	0.127	0.127	-0.035	0.169	0.103	0.115	-0.073
	(0.171)	(0.157)	(0.168)	(0.156)	(0.151)	(0.171)	(0.167)	(0.145)
$\ln(1 + R_m + Marr. \times R_f)$		0.557*		0.716		0.465*		-0.131
		(0.117)		(0.384)		(0.106)		(0.224)
$\ln(1 + e^{\Phi_m} + Marr. \times e^{\Phi_f})$			0.363				0.311	
			(0.411)				(0.410)	
$\frac{R_m}{2(1+R_m)^2}$				16.853				
				(10.417)				
$\frac{\ln(g^o)-E[\ln(g^o)]}{Var(\ln(g^o))}$				-0.461*				-0.765*
				(0.149)				(0.100)
Father some coll.					0.128	0.115	0.195	0.101
					(0.217)	(0.253)	(0.228)	(0.212)
Father coll+					-0.334	-0.677*	-0.447	-0.208
					(0.212)	(0.242)	(0.247)	(0.211)
Father's age					-0.016	0.017	-0.005	0.006
					(0.020)	(0.024)	(0.021)	(0.021)
$\frac{R_f(1+R_m)}{2(1+R_m+R_f)^2}$								8.015
								(5.819)
$\frac{R_m(1+R_f)}{2(1+R_m+R_f)^2}$								-13.710*
								(6.630)
$\frac{R_m+R_f}{2(1+R_m+R_f)^2}$								0.608
								(8.411)
Constant	1.281	-1.250	-0.075	-2.930	1.380	-0.235	0.140	1.570
	(0.765)	(0.879)	(1.714)	(1.697)	(0.769)	(0.911)	(1.711)	(1.166)
R-squared	0.175	0.385	0.189	0.469	0.131	0.277	0.130	0.510
Sample size	120	94	112	94	227	155	198	155

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. Columns 1-4 report results for single mothers and columns 5-8 report results for two-parent households. * significant at 0.05 level.

Table F-16: OLS estimates for child care/goods relative demand by child age

	(1) Ages 5-8	(2) Ages 9-12	(3) Ages 5-8	(4) Ages 9-12	(5) Ages 5-8	(6) Ages 9-12
$\ln(\tilde{P}_{c,t})$	0.332	0.874*	0.372	0.637	0.156	0.849*
	(0.265)	(0.331)	(0.288)	(0.349)	(0.280)	(0.339)
Married	1.033	0.555	0.933	-1.348	0.173	0.509
	(0.715)	(0.982)	(0.824)	(1.105)	(0.777)	(1.060)
Child's age	-0.235*	0.085	0.023	0.073	-0.135	0.054
	(0.096)	(0.096)	(0.109)	(0.109)	(0.110)	(0.114)
Mother some coll.	0.018	-0.116	0.109	0.098	0.005	0.016
	(0.217)	(0.247)	(0.232)	(0.276)	(0.226)	(0.264)
Mother coll+	-0.524*	-0.031	-0.213	0.168	-0.423	0.060
	(0.217)	(0.279)	(0.236)	(0.302)	(0.224)	(0.306)
Mother's age	0.015	-0.014	0.004	-0.022	0.007	-0.010
	(0.019)	(0.022)	(0.021)	(0.024)	(0.019)	(0.023)
Marr x Father some coll.	0.080	0.165	-0.117	0.265	0.173	0.221
	(0.259)	(0.350)	(0.283)	(0.392)	(0.268)	(0.356)
Marr x Father coll+	-0.214	-0.746*	-0.417	-1.100*	-0.431	-0.809*
	(0.269)	(0.372)	(0.293)	(0.422)	(0.307)	(0.403)
Marr x Father's age	-0.025	-0.008	-0.021	0.029	-0.008	-0.010
	(0.019)	(0.025)	(0.022)	(0.028)	(0.020)	(0.026)
Mother white	-0.226	-0.444	-0.202	-0.422	-0.054	-0.473*
	(0.177)	(0.229)	(0.185)	(0.242)	(0.188)	(0.237)
Num children ages 0-5	0.002	-0.079	0.370	0.074	0.052	0.010
	(0.175)	(0.262)	(0.194)	(0.288)	(0.191)	(0.292)
Num of children	-0.007	0.177	0.033	0.207	-0.077	0.262
	(0.147)	(0.178)	(0.149)	(0.188)	(0.154)	(0.186)
$\ln(1 + R_{m,t} + Marr \cdot R_{f,t})$			0.471*	0.474*		
			(0.098)	(0.128)		
$\ln(1 + e^{\Phi_{m,t}} + Marr \cdot e^{\Phi_{f,t}})$					0.582	0.020
					(0.371)	(0.492)
Constant	1.834	-1.005	-1.307	-1.659	0.037	-1.045
	(0.936)	(1.235)	(1.169)	(1.341)	(1.574)	(2.007)
R-squared	0.143	0.104	0.317	0.261	0.123	0.118
Residual sum of squares	208.800	225.446	108.628	135.248	170.075	196.242
F-test equality by age (p-value)	0.349		0.411		0.474	
Sample Size	186	161	130	119	163	147

Notes: Sample includes families in 2002 or 2007 PSID-CDS with children ages 5-12 and only 1-2 children ages 12 and under. * significant at 0.05 level.

Table F-17: Joint GMM Estimation of Relative Demand Moments

	$\epsilon_{\tau,g}$		$\epsilon_{Y,H}$		Correl. residuals	
	(1)	(2)	(1)	(2)	(1)	(2)
	0.20	0.37	0.51	0.76	0.88	0.88
	(0.05)	(0.17)	(0.08)	(0.09)		
	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_Y$: Childcare	
	(1)	(2)	(1)	(2)	(1)	(2)
Constant	8.30	4.30	4.09	3.35	-1.19	-1.46
	(1.94)	(0.62)	(1.27)	(0.76)	(0.40)	(0.28)
Single	0.28	0.11	-	-	0.62	0.64
	(0.38)	(0.15)			(0.21)	(0.14)
Mother some coll.	-0.44	-0.13	-	-	0.00	-0.05
	(0.45)	(0.28)			(0.19)	(0.13)
Mother coll+	-1.78	-0.76	-	-	-0.20	-0.28
	(0.76)	(0.65)			(0.19)	(0.13)
Child's age	-0.69	-0.34	-0.47	-0.24	-0.07	-0.04
	(0.18)	(0.15)	(0.18)	(0.14)	(0.03)	(0.02)
Num. of Children (0-5)	0.34	0.16	0.59	0.29	0.09	0.08
	(0.29)	(0.18)	(0.41)	(0.26)	(0.12)	(0.08)
Type 2	-1.14	-0.49	-	-	0.03	0.01
	(0.59)	(0.46)		-	(0.29)	(0.20)
Type 3	-2.46	-1.08	-	-	-0.04	-0.10
	(0.94)	(0.86)		-	(0.30)	(0.21)
Father some coll.	-	-	-0.83	0.04	-0.53	-0.41
			(0.66)	(0.25)	(0.23)	(0.16)
Father coll+	-	-	-1.07	-0.41	-0.06	0.01
			(0.73)	(0.45)	(0.25)	(0.17)

Notes: Specification (1) uses own-relative prices as instruments in each moment condition. Specification (2) uses predicted wages by occupation and state as an instrument for Mothers' and Fathers' wages. The column "Correl. residuals" reports the p-value from a correlation test of the relative demand residuals described in Appendix E.3. The function f is specified as: $f = \left[\left(\tilde{a}_{m,i,t}\tau_{m,i,t}^{\rho} + \tilde{a}_{f,i,t}\tau_{f,i,t}^{\rho} + (1-\tilde{a}_{m,i,t}-\tilde{a}_{f,i,t})g_{i,t}^{\rho}\right)^{\gamma/\rho}(1-\tilde{a}_{Yc,i,t}) + \tilde{a}_{Yc,i,t}Y_{c,i,t}^{\gamma}\right]^{1/\gamma}$ with $\tilde{a}_{j,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_j)}{1+\exp(Z_{i,t}\tilde{\phi}_m)+\exp(Z_{i,t}\tilde{\phi}_f)}$, $j \in \{m, f\}$ and $\tilde{a}_{Yc,i,t} = \frac{\exp(Z_{i,t}\tilde{\phi}_Y)}{1+\exp(Z_{i,t}\tilde{\phi}_Y)}$.

Table F-18: Joint GMM Estimation – No Borrowing/Saving ($\kappa = 1$)

	$\epsilon_{\tau,g}$				$\epsilon_{Y,H}$				δ_1				δ_2			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
	0.31	0.26	0.20	0.18	0.52	0.45	0.50	0.50	0.13	0.14	0.12	0.12	0.92	0.92	0.93	0.93
	(0.05)	(0.05)	(0.05)	(0.05)	(0.08)	(0.08)	(0.08)	(0.09)	(0.05)	(0.05)	(0.04)	(0.04)	(0.01)	(0.01)	(0.01)	(0.01)
	$\tilde{\phi}_m$: Mother's Time				$\tilde{\phi}_f$: Father's Time				$\tilde{\phi}_Y$: Childcare				$\overline{\phi}_\theta$: TFP			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Constant	5.15	6.25	8.28	13.28	3.28	3.38	4.01	4.12	-1.17^{+}	−1.20	−1.19	−1.42	-1.53	-1.76	-1.46	-0.69
	(0.70)	(0.99)	(1.93)	(4.02)	(0.76)	(0.89)	(1.24)	(1.41)	(0.32)	(0.44)	(0.41)	(0.61)	(0.52)	(0.54)	(0.43)	(0.32)
Single	0.14	0.12	0.03	0.10	-	-	-	-	0.51^{+}	0.50	0.56	0.59	-0.11	-0.11	-0.10	-0.08
	(0.24)	(0.28)	(0.37)	(0.41)	-	-	-	-	(0.20)	(0.24)	(0.21)	(0.21)	(0.06)	(0.07)	(0.07)	(0.07)
Mother some coll.	−0.19	-	−0.33	−0.38	-	-	-	-	0.04	-	−0.01	0.04	0.01	-	-0.02	-0.01
	(0.28)	-	(0.44)	(0.50)	-	-	-	-	(0.19)	-	(0.20)	(0.20)	(0.07)	-	(0.07)	(0.07)
Mother coll+	−0.85	-	−1.57	−1.75	-	-	-	-	−0.22	-	−0.26	−0.23	-0.05	-	-0.09	-0.11
	(0.36)	-	(0.71)	(0.84)	-	-	-	-	(0.18)	-	(0.19)	(0.19)	(0.10)	-	(0.11)	(0.11)
Child's age	−0.38	−0.44	−0.58	−0.66	−0.27	−0.32	−0.46	−0.51	-0.06^{+}	−0.07	−0.06	−0.06	-0.01	-0.01	-0.02	-0.03
	(0.08)	(0.10)	(0.17)	(0.22)	(0.09)	(0.11)	(0.17)	(0.21)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
Num. of children 0-5	0.32	0.40	0.48	0.61	0.40	0.44	0.62	0.79	0.11	0.12	0.10	0.10	0.13	0.14	0.14	0.12
	(0.19)	(0.22)	(0.30)	(0.36)	(0.25)	(0.29)	(0.42)	(0.49)	(0.12)	(0.14)	(0.12)	(0.13)	(0.05)	(0.05)	(0.05)	(0.05)
Type 2	-	−0.80	−1.24	-	-	-	-	-	-	0.14	0.10	-	-	0.13	0.13	-
	-	(0.40)	(0.61)	-	-	-	-	-	-	(0.34)	(0.31)	-	-	(0.10)	(0.10)	-
Type 3	-	−1.91	−2.76	-	-	-	-	-	-	0.09	0.04	-	-	-0.17	-0.16	-
	-	(0.60)	(1.01)	-	-	-	-	-	-	(0.34)	(0.31)	-	-	(0.15)	(0.15)	-
μ_k	-	-	-	−3.03	-	-	-	-	-	-	-	0.12^{+}	-	-	-	-0.37
	-	-	-	(1.23)	-	-	-	-	-	-	-	(0.25)	-	-	-	(0.16)
Father some coll.	-	-	-	-	−0.19	−0.20	−0.68	−0.83	−0.67	−0.72	−0.68	−0.73	0.28	0.27	0.26	0.27
	-	-	-	-	(0.37)	(0.41)	(0.63)	(0.73)	(0.21)	(0.25)	(0.24)	(0.24)	(0.09)	(0.09)	(0.09)	(0.09)
Father coll+	-	-	-	-	−0.50	−0.61	−1.04	−1.14	−0.00	0.04	−0.01	−0.03	0.09	0.07	0.06	0.08
	-	-	-	-	(0.42)	(0.49)	(0.72)	(0.83)	(0.24)	(0.28)	(0.25)	(0.25)	(0.08)	(0.09)	(0.08)	(0.08)
Year = 2002	-	-	-	-	-	-	-	-	-	-	-	-	0.21	0.22	0.20	0.21
	-	-	-	-	-	-	-	-	-	-	-	-	(0.07)	(0.07)	(0.07)	(0.07)

Notes: The superscript $^{+}$ indicates, using a Lagrange Multiplier test, rejection at 5% significance of the null hypothesis that an individual parameter enters identically in the demand and production moments. See Appendix E.3 for more details.

Table F-19: Joint GMM Estimation – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$				$\epsilon_{Y,H}$				δ_1				δ_2			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
	0.32	0.26	0.20	0.18	0.52	0.44	0.49	0.50	0.06	0.08	0.08	0.07	0.93	0.93	0.93	0.94
	(0.05)	(0.05)	(0.05)	(0.05)	(0.08)	(0.08)	(0.08)	(0.09)	(0.04)	(0.04)	(0.04)	(0.04)	(0.01)	(0.01)	(0.01)	(0.01)
	ϕ_m: Mother's Time				ϕ_f: Father's Time				ϕ_Y: Childcare				$\overline{\phi}_\theta$: TFP			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Constant	5.07	6.32	8.30	13.18	3.26^+	3.49^+	4.07	4.17	-1.17^+	-1.21^+	-1.19^+	-1.45^+	-0.78	-1.19	-1.06	-0.49
	(0.68)	(1.02)	(1.96)	(3.96)	(0.74)	(0.90)	(1.26)	(1.41)	(0.32)	(0.44)	(0.41)	(0.61)	(0.46)	(0.49)	(0.40)	(0.30)
Single	0.13	0.11^+	-0.00^+	0.07^+	-	-	-	-	0.52^+	0.52	0.57	0.60^+	-0.08	-0.07	-0.07	-0.06
	(0.24)	(0.28)	(0.37)	(0.41)	-	-	-	-	(0.20)	(0.24)	(0.21)	(0.21)	(0.06)	(0.06)	(0.06)	(0.06)
Mother some coll.	−0.19	-	−0.33	−0.39	-	-	-	-	0.04	-	−0.01	0.04	0.07	-	0.03	0.05
	(0.27)	-	(0.44)	(0.49)	-	-	-	-	(0.19)	-	(0.20)	(0.20)	(0.06)	-	(0.06)	(0.06)
Mother coll+	−0.83	-	−1.59	−1.74	-	-	-	-	−0.22	-	−0.27	−0.23	0.06	-	0.01	0.01
	(0.35)	-	(0.72)	(0.83)	-	-	-	-	(0.18)	-	(0.19)	(0.19)	(0.08)	-	(0.10)	(0.10)
Child's age	−0.36	−0.45	−0.58	−0.65	-0.27^+	-0.34^+	-0.47^+	-0.51^+	-0.06^+	-0.06^+	-0.06^+	-0.06^+	-0.02	-0.02	-0.02	-0.03
	(0.08)	(0.10)	(0.17)	(0.21)	(0.08)	(0.11)	(0.17)	(0.21)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)
Num. of children 0-5	0.35	0.44	0.52	0.64	0.41	0.46	0.64	0.80	0.10	0.13	0.10	0.10	0.14	0.16	0.15	0.14
	(0.18)	(0.23)	(0.31)	(0.36)	(0.25)	(0.30)	(0.42)	(0.49)	(0.12)	(0.14)	(0.12)	(0.13)	(0.05)	(0.05)	(0.05)	(0.05)
Type 2	-	−0.81	−1.25	-	-	-	-	-	-	0.12	0.08	-	-	0.23	0.23	-
	-	(0.41)	(0.61)	-	-	-	-	-	-	(0.34)	(0.31)	-	-	(0.08)	(0.08)	-
Type 3	-	−1.94	-2.78^+	-	-	-	-	-	-	0.08	0.03	-	-	0.01	0.02	-
	-	(0.61)	(1.03)	-	-	-	-	-	-	(0.34)	(0.31)	-	-	(0.12)	(0.12)	-
μ_k	-	-	-	-3.00^+	-	-	-	-	-	-	-	0.12^+	-	-	-	-0.17
	-	-	-	(1.21)	-	-	-	-	-	-	-	(0.25)	-	-	-	(0.14)
Father some coll.	-	-	-	-	−0.17	−0.23	−0.70	−0.84	−0.66	−0.71	−0.67	−0.73	0.33	0.34	0.30	0.31
	-	-	-	-	(0.36)	(0.42)	(0.64)	(0.72)	(0.21)	(0.25)	(0.24)	(0.24)	(0.08)	(0.08)	(0.08)	(0.08)
Father coll+	-	-	-	-	−0.47	−0.63	−1.03	−1.11	−0.00	0.06	0.00	−0.03	0.12	0.11	0.09	0.11
	-	-	-	-	(0.41)	(0.50)	(0.73)	(0.82)	(0.24)	(0.29)	(0.25)	(0.25)	(0.08)	(0.08)	(0.08)	(0.08)
Year = 2002	-	-	-	-	-	-	-	-	-	-	-	-	0.14	0.16	0.14	0.15
	-	-	-	-	-	-	-	-	-	-	-	-	(0.06)	(0.06)	(0.06)	(0.06)

Notes: The superscript $^+$ indicates, using a Lagrange Multiplier test, rejection at 5% significance of the null hypothesis that an individual parameter enters identically in the demand and production moments. See Appendix E.3 for more details.

Table F-20: Joint GMM Estimation Relaxing Some Parameters Across Relative Demand and Production – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$		$\epsilon_{Y,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.18	-	0.51	-	0.12	0.92	6.52
	(0.05)	-	(0.09)	-	(0.05)	(0.04)	(0.26)

	ϕ_m: Mother's Time		ϕ_f: Father's Time		ϕ_Y: Childcare		ϕ_θ: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	8.87	-	4.19	-	-1.19	181.80	-1.71
	(2.23)		(1.38)		(0.40)	(51370778.47)	(0.75)
Single	0.02	3.09	-	-	0.60	-	0.08
	(0.40)	(7.19)			(0.20)		(0.21)
Type 2	-1.36	-	-	-	0.08	-	0.20
	(0.67)				(0.29)		(0.13)
Type 3	-3.01	0.64	-	-	0.04	-	0.13
	(1.15)	(5.38)			(0.30)		(0.26)
Mother some coll.	-0.39	-	-	-	-0.02	-	0.03
	(0.48)				(0.19)		(0.09)
Mother coll+	-1.76	-	-	-	-0.30	-	-0.01
	(0.81)				(0.18)		(0.14)
Child's age	-0.64	-	-0.50	-2.14	-0.07	-15.07	-0.00
	(0.20)		(0.20)	(772359423.69)	(0.03)	(4290576.05)	(0.03)
Num. of children 0-5	0.52	-	0.69	-	0.09	-	0.16
	(0.33)		(0.46)		(0.12)		(0.07)
Father some coll.	-	-	-0.78	-	-0.65	-	0.27
			(0.70)		(0.23)		(0.11)
Father coll+	-	-	-1.13	-	-0.02	-	0.06
			(0.79)		(0.24)		(0.09)
Year = 2002	-	-	-	-	-	-	0.17
							(0.10)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.3 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses *except* for the distance metric, which reports a p-value.

Table F-21: Joint GMM Estimation Allowing Time Productivity Share for Mothers to Differ Across Relative Demand and Production – No Borrowing/Saving ($\kappa = 1$)

	$\epsilon_{\tau,g}$		$\epsilon_{Y,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.20	-	0.49	-	0.13	0.92	0.33
	(0.05)	-	(0.08)	-	(0.05)	(0.02)	(0.56)
	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_Y$: Childcare		ϕ_θ: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	8.38	10.50	4.05	-	-1.18	-	-1.68
	(1.98)	(4.99)	(1.27)		(0.41)		(0.69)
Single	0.01	-	-	-	0.57	-	-0.06
	(0.37)				(0.21)		(0.07)
Type 2	-1.26	-	-	-	0.10	-	0.19
	(0.61)				(0.31)		(0.14)
Type 3	-2.79	-	-	-	0.04	-	-0.04
	(1.03)				(0.31)		(0.26)
Mother some coll.	-0.34	-	-	-	-0.01	-	0.00
	(0.44)				(0.20)		(0.08)
Mother coll+	-1.60	-	-	-	-0.28	-	-0.02
	(0.73)				(0.19)		(0.16)
Child's age	-0.59	-	-0.47	-	-0.06	-	-0.00
	(0.18)		(0.18)		(0.03)		(0.05)
Num. of children 0-5	0.45	-	0.60	-	0.09	-	0.13
	(0.30)		(0.42)		(0.12)		(0.05)
Father some coll.	-	-	-0.69	-	-0.69	-	0.23
			(0.64)		(0.24)		(0.10)
Father coll+	-	-	-1.05	-	-0.00	-	0.07
			(0.73)		(0.25)		(0.09)
Year = 2002	-	-	-	-	-	-	0.19
							(0.07)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.3 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses *except* for the distance metric, which reports a p-value.

Table F-22: Joint GMM Estimation Allowing Time Productivity Share for Mothers to Differ Across Relative Demand and Production – Unconstrained ($\kappa = 0$)

	$\epsilon_{\tau,g}$		$\epsilon_{Y,H}$		δ_1	δ_2	$2N(Q_N - \tilde{Q}_N)$
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-	-	-
	0.20	-	0.49	-	0.08	0.93	0.26
	(0.05)	-	(0.08)	-	(0.04)	(0.01)	(0.61)

	$\tilde{\phi}_m$: Mother's Time		$\tilde{\phi}_f$: Father's Time		$\tilde{\phi}_Y$: Childcare		ϕ_θ: TFP
	Rel. Dem.	Prod.	Rel. Dem.	Prod.	Rel. Dem.	Prod.	-
Constant	8.40	10.05	4.08	-	-1.19	-	-1.18
	(2.00)	(7.40)	(1.28)		(0.41)		(0.64)
Single	-0.00	-	-	-	0.57	-	-0.05
	(0.38)				(0.21)		(0.07)
Type 2	-1.27	-	-	-	0.08	-	0.26
	(0.62)				(0.31)		(0.16)
Type 3	-2.81	-	-	-	0.03	-	0.07
	(1.04)				(0.31)		(0.29)
Mother some coll.	-0.34	-	-	-	-0.01	-	0.04
	(0.45)				(0.20)		(0.08)
Mother coll+	-1.62	-	-	-	-0.27	-	0.05
	(0.73)				(0.19)		(0.19)
Child's age	-0.59	-	-0.47	-	-0.06	-	-0.01
	(0.18)		(0.18)		(0.03)		(0.05)
Num. of children 0-5	0.52	-	0.65	-	0.10	-	0.15
	(0.31)		(0.42)		(0.12)		(0.05)
Father some coll.	-	-	-0.73	-	-0.67	-	0.29
			(0.65)		(0.24)		(0.09)
Father coll+	-	-	-1.05	-	-0.00	-	0.09
			(0.74)		(0.25)		(0.08)
Year = 2002	-	-	-	-	-	-	0.14
							(0.06)

Notes: This table reports GMM estimates where some parameters are allowed to differ between those determining relative input demand (Rel. Dem.) and those determining actual skill production (Prod.). See Appendix E.3 for more details. The distance metric, $2N(Q_N - \tilde{Q}_N)$, is the difference between the optimally weighted GMM criterion at the restricted estimates and its value at the relaxed estimates. It has a χ^2 distribution with degrees of freedom equal to the number of constraints that are relaxed. Standard errors are indicated in parentheses *except* for the distance metric, which reports a p-value.