On the Individual Optimality of Economic Integration*

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September 2010
Comments Welcome

Abstract

Which countries find it individually optimal to form an economic union? We emphasize the risk-sharing benefits of economic integration. We consider an endowment world economy model, where international financial markets are incomplete and contracts not enforceable. A union is an arrangement that solves both the market incompleteness and the lack of enforcement problems among member countries. The union as a whole still faces these frictions when trading in the world economy. We uncover conditions on the initial income and net foreign assets of potential union members such that forming a union is welfare-improving over standing alone in the world economy. Consistently with evidence we gather on economic integration, our model predicts that economic unions (i) occur relatively infrequently, and (ii) are more likely to emerge among homogeneous countries, and (iii) among rich countries.

Keywords: Incomplete markets, endogenous borrowing constraints, risk sharing, economic integration.

JEL Codes: F15, F34, F36, F41.

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*We thank Silvia Gonçalves, Per Krusell, Mark Wright, and seminar attendants at the Atlanta Fed, Queen’s University, the Washington University in St.Louis/St.Louis Fed, Université de Montréal, University of Windsor, 2010 Midwest Macro Meetings in East Lansing, 2009 CEA meetings in Toronto, 2009 Portuguese Economic Journal meetings in Madeira, and 2009 SED meetings in Istanbul for helpful comments. Castro acknowledges financial support from SSHRC.

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1 Introduction

Which countries find it individually optimal to form an economic union? We emphasize a particular motivation for economic integration: improving risk sharing. An economic union is a small-scale arrangement where partners are better able to cope with the frictions that limit risk-sharing in the world economy.

We consider an initial situation in which countries are sitting in the world economy with very limited possibilities to sharing idiosyncratic endowment risk. Risk sharing is limited by two frictions. First, markets are incomplete since countries may only trade a non-contingent bond. Second, international lending contracts are not legally enforceable. At any time, a country may choose to repudiate its foreign debt. The sanction for doing so is the permanent exclusion from future trade in world markets. Our world economy model is a variant of Clarida (1990) and Huggett (1993), featuring self-enforcing borrowing limits along the lines of Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000). Versions of this setup have been studied previously in different contexts by Zhang (1997) and Krueger and Perri (2006).¹

We then consider the possibility that a pair of countries selected at random from the world economy is suddenly offered the possibility of forming an economic union. A union, by assumption, is an arrangement which solves both the market incompleteness and the lack of enforcement problems among member countries. The union as a whole, however, still faces these frictions when trading in world markets. Since the endowment risk facing union members cannot be fully diversified away, they still have an interest in trading with the rest of the world. We focus on setting where union members coordinate their international borrowing and lending and default decisions, as if these decisions were taken by a central union authority.

The key trade-off our model emphasizes about union formation, from the perspective of each individual country, is the following. There are two benefits from economic integration. First, forming a union improves risk-sharing opportunities among member countries. Second, a union allows for poor partners to use the rich partners’ credit lines. The latter is a benefit for poor partners only. There are also two costs of economic integration. First, borrowing limits become tighter, since defaulting on international debt becomes less costly for union partners. This happens because union partners may still share risk upon default. Second, since poor partners may benefit

¹See Abraham and Carceles-Poveda (2009) and Bai and Zhang (2010) for variants with capital accumulation. See also Castro (2005) for a variant with capital accumulation and endogenous but ad-hoc borrowing constraints.
from the rich partner’s credit limit, this generates a negative externality: rich partners will find themselves more often borrowing-constrained in a union compared to standing alone in the world economy.

Our model generates not only aggregate benefits, but also aggregate costs of economic integration. In addition, our model also generates disagreement about union formation, and the disagreement is the largest when the partners are more heterogeneous. These two ingredients provide a potential explanation for three seemingly puzzling empirical observations on economic integration: (i) deep economic integration is relatively rare, and when it does take place it tends to feature (ii) relatively homogeneous partners, and (iii) relatively richer partners. Our paper provides some empirical evidence documenting these regularities.

These observations are puzzling because, under a very broad set of circumstances, economic theory would imply that economic integration should happen often, particularly among heterogeneous partners. For example, this would be the case for capital market integration in the neoclassical growth model, or goods market integration in either the Heckscher-Ohlin or the Ricardian models of trade.²

Our framework provides a very parsimonious explanation for these puzzling observations. Economic unions may not be formed if either the aggregate costs of economic integration are too large, or if there is disagreement among partners. Unions are unlikely to be formed among heterogeneous partners, since rich partners suffer from a negative externality imposed by poor partners. Finally, unions are also more likely to be formed among relatively rich partners because this lowers the likelihood of either country being borrowing-constrained in the future, and thus the effect of the negative externality.

This paper is related to a vast literature that has attempted to estimate the welfare gains from full international risk-sharing. This literature includes papers such as Cole and Obstfeld (1991), Backus, Kehoe, and Kydland (1992), Obstfeld (1994b,a), van Wincoop (1994, 1999), Mendoza

²Union formation in intra-industry trade models, emphasizing scale economies and a taste for variety, have been analyzed in a static setting by Krugman (1991), Frankel, Stein, and Wei (1995), Frankel (1997) and Baier and Bergstrand (2004). This type of model emphasizes size as a determinant of union formation: the larger and the more similar the partners’ market sizes, the larger the gains from goods market integration. Larger unions profit more from scale economies, and size homogeneity lowers the losses from trade diversion. While Baier and Bergstrand (2004) find empirical support for these implications, our data also suggests that, beyond market size, the level and the dispersion in partner wealth matters for economic integration. Differently from this literature, our paper focuses on heterogeneity in per capita incomes and net foreign assets over GDP.
(1995), Tesar (1995), Lewis (2000), and Athanasoulis and van Wincoop (2000). The typical exercise computes the average gain across countries of going from financial market autarky to complete markets, and entirely eliminating idiosyncratic country risk. Although the range of estimated welfare gains is large, the gains are still positive in nearly all the papers. The sole exception is Devereux and Smith (1994), who like this paper also model costs of sharing risk. In their case, sharing risk lowers precautionary saving, which lowers output growth and might lower welfare. We emphasize instead the tightening of credit constraints, and the negative externalities generated by poor union partners.

The present paper differs from this literature in several dimensions. First, beyond the magnitude of the welfare gains, this paper is mostly interested on their distribution across countries. Even if the average gains might be high, they can be very oddly distributed. If some countries actually experience a loss, as it is often the case in our model, risk sharing arrangements may not take place at all. This may explain the observed lack of international risk diversification, even in the presence of possibly large average welfare gains. Moreover, the main prediction of our model can be tested against the evidence, namely that feasible risk-sharing arrangements should occur among homogeneous and rich countries.

Second, this paper considers financial market integration as it typically takes place in the real world. That is, as voluntary arrangements among small sets of countries. Financially integrated countries are still unable to share risk with the rest of the world. Further, in our paper countries may save and self-insure in the absence of complete markets, whereas most of the literature abstracts from this feature. Our paper computes welfare gains from international risk-sharing that take these important features into account.

A recent paper that has also looked at potential risk sharing arrangements within small sets of countries is Imbs and Mauro (2008). Using actual data on the variance-covariance matrix of cross-country output growth, they uncover the number and configuration of countries that offer the best risk-sharing potential. Like in the rest of the international risk-sharing literature, they focus on going from autarky to complete markets, and do not feature neither costs of economic integration, nor a role for disagreement among partners. Their main finding is that most diversification gains are achieved in arrangements featuring a small (up to seven) number of countries, and in arrangements between highly volatile countries. As Imbs and Mauro (2008) recognize, a natural question is why we do not observe more arrangements of this type. They argue that this could be because unions might be particularly costly to sustain among volatile countries, since these also tend to have poor
contract enforcement institutions. While our framework abstracts from cross-country differences in output volatility, it does provide an explicit, alternative reason for why small-size arrangements may not be feasible, even in the face of large aggregate gains.

The paper is organized as follows. Section 2 presents some evidence about union formation. Section 3 presents the model of the world economy. Section 4 characterizes the union. Section 5 presents the results. Section 6 concludes. Appendix A provides some details about the data. Appendices B and C describe the decentralization of the union’s allocation and the numerical algorithm, respectively.

2 Empirical Evidence

We start by providing some empirical evidence on the role of wealth levels and wealth inequality for union formation. By wealth we mean both income \((y)\) and net foreign assets \((b)\), both variables being potentially relevant according to our formal model. Our approach is to run a probit-gravity regression to test whether wealth levels contribute positively, and wealth inequality negatively, for the probability of union formation. Our regression specification is a straightforward adaptation of those commonly used in the empirical trade literature to test predictions over bilateral trade flows (see Frankel and Romer (1999), Frankel and Rose (2002)), similar to Baier and Bergstrand (2004).

We consider:

\[
\text{Prob}\{\text{Union}_{ij} = 1|X_{ij}\} = \Phi(X'_{ij}\beta)
\]

with

\[
X'_{ij}\beta = \alpha_1 + \alpha_2 \ln(\text{dist})_{ij} + \alpha_3 \text{adj}_{ij}
+ (\alpha_4 + \alpha_4^a \text{adj}_{ij}) \ln(\text{pop}_i \times \text{pop}_j) + (\alpha_5 + \alpha_5^a \text{adj}_{ij}) \ln \left(\frac{\text{pop}_i}{\text{pop}_j}\right)
+ (\theta_1 + \theta_1^a \text{adj}_{ij}) \ln(y_i + y_j) + (\theta_2 + \theta_2^a \text{adj}_{ij}) \ln \left(\frac{y_i}{y_j}\right)
+ (\gamma_1 + \gamma_1^a \text{adj}_{ij}) \left(\frac{b_i}{y_i} + \frac{b_j}{y_j}\right) + (\gamma_2 + \gamma_2^a \text{adj}_{ij}) \left(\frac{b_i}{y_i} - \frac{b_j}{y_j}\right).
\]

The dependent variable is a dummy which gets the value of 1 if a union is formed between countries \(i\) and \(j\), and 0 otherwise. The regressors in the first two lines of the regression equation concern factors deemed to be important for union formation but absent from our theoretical framework. The last two lines concern wealth levels and wealth heterogeneity, the key determinants in
our theory.

We begin with the former set of regressors. We include two geographical factors commonly used in the gravity regression literature, the distance between the main economic centers of countries $i$ and $j$ ($\text{dist}_{ij}$), and a dummy variable capturing whether countries $i$ and $j$ share a common border ($\text{adj}_{ij}$). We also include overall size and a measure of heterogeneity in size, as potential determinants of union formation, where size is measured by population ($\text{pop}_i$). In particular, Baier and Bergstrand (2004) have found scale effects to be important for union formation, consistent with the predictions of a class of intra-industry trade models. In the last two lines, we include the overall income level of the country pair $(i,j)$, a measure of the inequality in incomes between the two countries, and similarly for net foreign assets over income. We make the contribution of wealth levels and wealth inequality for union formation contingent upon whether countries share a border, and similarly for size. This specification finds a parallel in Frankel and Romer (1999).

To implement our regression analysis, we combine a variety of data sets. From version 6.3 of the Penn World Tables (Heston, Summers, and Aten, 2009) we obtain our measure of income (real GDP per capita) and population. We obtain net foreign asset positions from Lane and Milesi-Ferretti (2007). We consider real GDP and nominal net foreign assets over nominal GDP averaged over five years (2000-2004) as our regressors, to prevent high frequency variation in these variables from affecting our results.

Our geographical data comes from Frankel and Rose (2002), and our union dummy is obtained from a comprehensive data set assembled by Baier and Bergstrand (2009). Based on information from the World Trade Organization, among other sources, this data set provides information on which countries are engaged in any kind of regional trade arrangement in any given year. The regional trade arrangements range from Preferential Trade Arrangements, to Free Trade Areas like NAFTA, to Economic Unions like the European Union. For reasons that will become apparent when we model unions in Section 4, we restrict our empirical definition of unions only to those arrangements characterized by a sufficiently deep level of economic integration. In particular, we do not consider Free Trade Areas like NAFTA as a union. This is because members of Free Trade Areas may set independent tariff policies vis-a-vis non-members, making it in our view inappropriate to think about them as a block. Our empirical definition of unions therefore includes Custom Unions (no trade barriers between members, common barriers vis-a-vis non-members), Common Markets (custom unions featuring free capital and labor mobility between members), and Economic Unions (common markets featuring harmonization of economic policy, namely fiscal and monetary). We
present regression results for different definitions of economic union, the results being generally robust across them.

We focus on a single cross-section of 136 countries in the year 2004. The year is the most recent one in the Baier and Bergstrand (2009) data set, and the number of countries is the maximum given the available data in 2004. We then consider all possible country pairings from this set. We assign the value of 1 to the union dummy if a particular country pair was part of a union in 2004, and 0 otherwise. Given the available geographical data, we end up with 6629 country pairings.

We report in Table 1 our estimated average marginal effects, conditional on either value for the common border dummy.

As expected, our results support a negative effect of distance on the probability of union formation. Regarding scale, the results are somewhat inconsistent with Baier and Bergstrand (2004), in the sense that scale tends to be detrimental to union formation, except for sufficiently deep unions, and conditional on countries not sharing a common border. However, like in Baier and Bergstrand (2004), inequality in scale is generally detrimental to union formation.

We now turn to the variables that are more relevant us. The evidence supports the view that the larger the partner’s combined incomes, the higher the probability of union formation, especially among non-adjacent countries. Income inequality is always clearly detrimental to union formation, and similarly for inequality in net foreign assets over GDP. The combined level of net foreign assets over GDP tends instead to detrimental for union formation, except for customs unions sharing a border. The only exception is for countries sharing a border and customs unions or deeper arrangements.

We take these results to support the broad view that, even when controlling for geographical factors and scale effects, wealth levels contribute positively, and wealth inequality contributes negatively to union formation.

4We treat newly-formed and continuing unions in 2004 both as instances of union formation, in line with Baier and Bergstrand (2009). This is a caveat of our empirical analysis since, in reality, there is a likely bias towards the status-quo. That is, everything else constant, existing unions are more likely to continue than new unions to form. Unfortunately, the extremely small number of newly-formed unions in any given year prevents us from concentrating only on new unions.
Table 1: Wealth, inequality, and union formation

<table>
<thead>
<tr>
<th>Definition of Union: at least...</th>
<th>...Customs Union</th>
<th>...Common Market</th>
<th>...Economic Union</th>
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<td>Distance</td>
<td>adj=0</td>
<td>adj=1</td>
<td>adj=0</td>
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<tr>
<td></td>
<td>−0.038 (0.000)</td>
<td>−0.037 (0.000)</td>
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<tr>
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<td>−0.023 (0.000)</td>
<td>−0.020 (0.000)</td>
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<td>Population Size</td>
<td>adj=0</td>
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<td>adj=0</td>
</tr>
<tr>
<td></td>
<td>−0.002 (0.007)</td>
<td>−0.001 (0.688)</td>
<td>−0.006 (0.000)</td>
</tr>
<tr>
<td></td>
<td>0.004 (0.000)</td>
<td>−0.001 (0.488)</td>
<td>−0.02 (0.042)</td>
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<tr>
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<td>0.004 (0.181)</td>
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<td>−0.004 (0.181)</td>
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<tr>
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<td>0.006 (0.964)</td>
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</table>

Notes: Huber-White robust p-values in parenthesis, computed by the delta method.

3 World economy

3.1 Model

Consider a world economy composed of a continuum of small open economies of measure one. Countries are identical ex-ante, and differ ex-post due to idiosyncratic endowment risk. Each period, a country receives an endowment of a non-storable consumption good. The endowment evolves over time according to a Markov chain with a finite number of states in the set $Y$. We denote by $y^t = \{y_s, y_{s+1}, \ldots, y_t\}$ the sequence of events from the initial time period $s < 0$ up to and including period $t$, and by $\pi(y^t)$ the probability of such sequence. The initial event $y^s = y_s$ is
given and $\pi(y^t) = 1$. We denote by $\pi(y^t|y^\tau)$ the probability of $y^t$ conditional on $y^\tau$ where $\tau \leq t$, and by $y^\tau \leq y^t$ the sequence $y^\tau$ which is a sub-root of $y^t$. We assume a law of large numbers holds in the cross-section of countries, which means there is no aggregate uncertainty.

Each country is populated by an infinitely-lived representative agent with preferences:

$$
\sum_{t=s}^{\infty} \sum_{y^t \in Y^{t+1}} \beta^t \pi(y^t) u(c(y^t)), \tag{3.1}
$$

where $\beta \in (0, 1)$ is the subjective discount factor. The instantaneous utility $u$ is increasing, strictly concave, and satisfies the Inada conditions: $\lim_{c \to 0} u'(c) = +\infty$ and $\lim_{c \to +\infty} u'(c) = 0$.

Countries cannot completely pool their income risk on world financial markets for two reasons. First, markets are incomplete: the menu of assets is exogenously restricted to a non-contingent one-period bond. A country’s resource constraint is

$$
c(y^t) + b(y^t) = y_t + (1 + r) b(y^{t-1}), \tag{3.2}
$$

where $b(y^t)$ is the demand for foreign bonds and $r$ is the (time-invariant) world interest rate.

The second friction is that international lending contracts are imperfectly enforceable. At any time, a country is free to repudiate its foreign debt, the penalty being the permanent exclusion from any future trade. A country that contemplates debt repudiation faces a trade-off between current and future utility: defaulting implies higher current consumption, at a cost of lower future utility due to living in autarky. International lending contracts are self-enforcing, in the sense that borrowing countries always find the cost of repudiation larger than the benefit, and they always choose to repay. That is, allocations satisfy the following participation constraint:

$$
\sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi(y^\tau|y^t) u(c(y^\tau)) \geq V_{aut}(y^t), \tag{3.3}
$$

where $V_{aut}(y^t)$ is the value of entering financial autarky after the history $y^t$. It is the lifetime utility derived from consuming one’s endowment each period from the history node $y^t$ onwards:

$$
V_{aut}(y^t) = \sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi(y^\tau|y^t) u((1 - \phi) y^\tau).
$$

The parameter $\phi \in [0, 1]$ is a direct output cost associated with default. Such additional default penalty has been considered in the literature, and it has been typically motivated as a way to capture production disruptions that occur because of lack of access to international markets. As in Arellano (2008), our motivation is mainly quantitative. Without such penalty, the extent of borrowing and lending in the model is much lower than in the data.
The representative agent chooses contingent plans for consumption and foreign assets to maximize lifetime utility (3.1) subject to the resource constraint (3.2), the enforcement constraint (3.3), and a no-Ponzi game condition:

\[ b(y') \geq -D, \]

where \( D \) is large enough that the constraint never binds in equilibrium.\(^4\)

### 3.2 Recursive competitive equilibrium

We solve for the stationary recursive competitive equilibrium with solvency constraints. The state of the economy is characterized by net foreign bond holdings \( b \) and by the current endowment \( y \).

The problem of each country admits the following recursive formulation (see Bai and Zhang (2010) for a formal proof):

\[
V(b, y) = \max_{c, b'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(b', y') \right\}
\]

subject to:

\[
c + b' = y' + (1 + r)b'
\]

\[
b' \geq b^W(y).
\]

The state-contingent borrowing constraint \( b^W \) is the debt level such that for every possible state next period, the country is weakly better-off by repaying:

\[
b^W(y) = \max_{y': \pi(y'|y) > 0} \{ b_{y'} : V(b_{y'}, y') = V_{aut}(y') \}.
\]

This constraint allows countries to borrow as much as possible while preventing them from defaulting in any possible state next period. The state-contingency arises only when there exist future states that cannot be reached from current state. We assume \( \pi(y'|y) > 0 \) for all \( y, y' \), so that \( b^W(y) = b^W \) for all \( y \in Y \).

The autarky value \( V_{aut} \) is the solution to the following functional equation:

\[
V_{aut}(y) = u ((1 - \phi) y) + \beta \sum_{y' \in Y} \pi(y'|y) V_{aut}(y').
\]

Let \( B \) be the set of net foreign bond levels, \( S = B \times Y \) the state-space, and \( \mathcal{A}_S \) the \( \sigma \)-Borel algebra of elements of \( S \). We are now ready to define the stationary recursive competitive equilibrium of the world economy.

\(^4\)Note that the enforcement constraint does not prevent countries from running Ponzi schemes: an agent running a Ponzi game would never default on its debt, since this would prevent him from continuing running the scheme.
Definition. A stationary recursive competitive equilibrium is given by decision rules \( c(b, y), b'(b, y) \), a value function \( V(b, y) \), a borrowing limit \( b^W \), an interest rate \( r \) and a distribution \( \Psi(b, y) \) of countries over \( S \) such that:

1. Given the world interest rate \( r \) and the borrowing limit \( b^W \), the decision rules solve the recursive problem \((P0)\) and \( V \) is the associated value function.

2. The borrowing limit \( b^W \) is not too tight, in the sense of satisfying equation \((3.5)\) for all \( y \).

3. The world credit market clears:
\[
\int_S b'(b, y) d\Psi(b, y) = 0.
\]

4. The decision rules and the transition matrix of the endowment process induce a probability distribution \( P \) over the state space, \( P : S \times A_S \rightarrow [0, 1] \), where:
\[
P((b, y); A) = \sum_{y' : (b', y') \in A} \pi(y'|y)
\]

is the probability of transiting from state \((b, y)\) to a state in the set \( A \).

5. The distribution \( \Psi \) is stationary and consistent with \( P \):
\[
\Psi(A) = \int_S P((b, y); A)d\Psi(b, y), \text{ for all } A \in A_S.
\]

3.3 Parameters and computation

Preferences are isoelastic:
\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad (3.7) \]
with a coefficient of relative risk aversion \( \sigma = 1.5 \). The subjective discount factor is selected so that the equilibrium world interest rate is 1%, yielding \( \beta = 0.9815 \).

The direct output penalty ensures that the cross-sectional standard deviation of the net foreign asset to GDP ratio equals 0.42, the average cross-sectional standard deviation obtained from the Lane and Milesi-Ferretti \((2007)\) data set - we focus on a balanced panel of 110 countries over the 1970-2004 period. This yields \( \phi = 0.00231 \), or about a 0.2 percent yearly drop in output during default.

The endowment process is obtained from estimating the empirical first-order autoregressive process on a panel of countries:
\[
\ln y_{it+1} = \mu_i + d_i + \rho \ln y_{it} + \varepsilon_{it+1},
\]
where $\varepsilon_{it+1}$ follows an i.i.d. $N(0, \sigma^2_\varepsilon)$. We include time dummies ($d_t$) to capture world business cycle effects. We estimate this process by pooling data on linearly detrended real output per capita from version 6.3 of the Penn World Tables (Heston, Summers, and Aten, 2009). We focus on a balanced panel of 111 countries over the 1960-2007 period. The point estimates of the key parameters are $\rho = 0.897$ and $\sigma_\varepsilon = 0.058$. In the model we normalize every country’s mean endowment to 1 and consider the common process

$$\ln y' = 0.897 \ln y + 0.058 \varepsilon', \nonumber$$

with $\varepsilon' \sim$ i.i.d. $N(0,1)$. This process is discretized into a 5-state Markov chain using Rouwenhorst’s (1995) procedure. The set of values for the endowment level $Y$ and the transition matrix $\Pi$ are reported in Table 2.

<table>
<thead>
<tr>
<th>Y</th>
<th>(y_t)</th>
<th>(y_{lm})</th>
<th>(y_m)</th>
<th>(y_{mh})</th>
<th>(y_h)</th>
</tr>
</thead>
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<tr>
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<td>1.140</td>
<td>1.300</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Pi$</th>
<th>0.809</th>
<th>0.176</th>
<th>0.014</th>
<th>$5 \times 10^{-4}$</th>
<th>$7 \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.044</td>
<td>0.817</td>
<td>0.132</td>
<td>0.007</td>
<td>$10^{-4}$</td>
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<tr>
<td></td>
<td>0.002</td>
<td>0.088</td>
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<td>0.002</td>
</tr>
<tr>
<td></td>
<td>$10^{-4}$</td>
<td>0.007</td>
<td>0.132</td>
<td>0.817</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>$7 \times 10^{-6}$</td>
<td>0.001</td>
<td>0.014</td>
<td>0.176</td>
<td>0.809</td>
</tr>
</tbody>
</table>

Table 2: Markov chain parameters

We briefly describe our numerical algorithm, the full details are provided in Appendix C.1. The outer loop solves for the interest rate that clears the world bond market. For given interest rate, we solve for debt limit functions which are not too tight, using the natural borrowing limit as the initial guess. Finally, for given interest rate and debt limit functions, we solve for the decision rules that solve the system of first-order conditions for the country’s problem.

### 4 Economic union

We now describe the process of union formation in the model. We assume the world economy is in steady-state. At time $t = 0$, and without anticipating it, a pair of countries sitting in the world economy is offered the possibility of forming a union. We pick these two countries from the ergodic state-space of the world economy’s stationary equilibrium. Each country is characterized by an initial state $(b_{i0}, y_{i0})$, $i = 1, 2$. We also assume that union formation is a once-and-for-all event, i.e. once a union is formed it cannot be dissolved in the future.
Within the union, we assume full enforcement, and complete financial markets. Since a union is comprised of a finite number of countries (in this case two), there is still some endowment risk that the union would like to diversify away with the rest of the world. We assume union members still have access to world financial markets under the same conditions as before, i.e. by trading on non-contingent bonds subject to enforcement constraints. The union is like a small country in the world economy.

We assume the existence of a central authority in the union that coordinates the international trade and default decisions. Since union members coordinate their default decisions, there is a single union-wide enforcement constraint that applies to both countries at the same time. If the union defaults, all its members are permanently excluded from world markets, but they may still share endowment risk among them.

The union’s endowment is determined by the realization of two independent and identically distributed endowment processes, one for each country. We denote it compactly by a two-dimensional vector \( \bar{\mathbf{y}}_t = (y_{1t}, y_{2t}) \in Y \times Y \), where each element \( y_{it} \in Y \) is country \( i \)'s endowment realization, \( i = 1, 2 \). With a slight abuse of notation, we also denote by \( \pi \) the transition probabilities for \( \bar{\mathbf{y}} \):

\[
\pi(\bar{\mathbf{y}}' | \bar{\mathbf{y}}) = \prod_{i=1}^{2} \pi(\mathbf{y}_i' | y_i),
\]

where the \( \pi(\mathbf{y}_i' | y_i) \)'s are displayed in Table 2.

### 4.1 Planner’s problem

The allocation within the union is constrained-efficient, and can be obtained by solving a benevolent planner’s problem. Although countries join the union with potentially different net foreign bond levels, only the aggregate net asset position matters for the planner’s problem. Let \( \bar{b}_0 = \sum_i b_{i0} \) and let \( \lambda_i \) be the weight the planner attaches to country \( i \). The planner’s problem is to solve for \( \{c_i(\bar{y}^t)\}_{i=1,2} \) and \( \bar{b}(\bar{y}^t) \), for all \( \bar{y}^t \), \( t \geq 0 \), which maximize the weighted sum of the union partners’ lifetime expected utilities.

---

5Note that completing markets may be achieved in a variety of ways, not just by increasing financial market sophistication. First, fiscal transfers in highly-integrated unions can achieve the same goal. Second, goods market liberalization may also complete markets. Cole and Obstfeld (1991) have shown that changes in terms of trade can go a long way towards insuring against idiosyncratic income risk; in some extreme cases trade in goods even provides all the necessary insurance, without the need for financial markets.
\[
\sum_{i=1}^{2} \lambda_i \sum_{t=0}^{\infty} \sum_{\gamma^t} \beta^t \pi(\gamma^t) u(c_i(\gamma^t))
\]

subject to the union-wide resource constraint

\[
\sum_{i} c_i(\gamma^t) + \bar{b}(\gamma^t) = \sum_{i} y_{it} + (1 + r)\bar{b}(\gamma^{t-1}),
\]

for all \(\gamma^t, t \geq 0\), to the union-wide enforcement constraint

\[
\sum_{i} \lambda_i \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi(\gamma^{\tau}|\gamma^t) u(c_i(\gamma^{\tau})) \geq W_{aut}^{U}(\gamma^t),
\]

for all \(\gamma^t, t \geq 0\), where

\[
W_{aut}^{U}(\gamma^t) = \max_{\{c_i(\gamma^{\tau})\}} \sum_{i} \lambda_i \sum_{\tau=t}^{\infty} \sum_{\gamma^t} \beta^{\tau-t} \pi(\gamma^{\tau}|\gamma^t) u(c_i(\gamma^{\tau}))
\]

subject to

\[
\sum_{i} c_i(\gamma^{\tau}) = (1 - \phi) \sum_{i} y_{i\tau}, \text{ for all } \gamma^{\tau}, \tau \geq t,
\]

for all \(\gamma^t, t \geq 0\), and subject also to a no-Ponzi game condition

\[
\bar{b}(\gamma^t) \geq -D,
\]

for all \(\gamma^t, t \geq 0\).

Apart from distributional issues, the planner’s problem is similar to the problem of a country standing alone in the world economy, the main difference being that, because the partners’ endowment processes are uncorrelated, the union faces an endowment process which is less volatile. Since markets are complete and contracts enforceable among union members, the lower aggregate endowment volatility translates into lower individual consumption volatility.

### 4.1.1 Reformulating the planner’s problem

Under isoelastic preferences, the union planner’s problem admits a simpler formulation which is very convenient. By Proposition 5 of Jeske (2006), aggregate borrowing and lending is independent of distributional issues. It follows that the planner’s problem may be decomposed into two steps. In the first step, the planner solves for the optimal borrowing and lending of the union assuming a single representative country facing the aggregate endowment. In the second step, the planner
redistributes the optimal aggregate consumption plan obtained from the first step among the two union partners.

Formally, the step 1 problem for the planner is

\[
\max_{c(y^t), b(y^t)} \sum_{t=0}^{\infty} \sum_{y^t} \beta^t \pi(y^t) u(c(y^t))
\]

subject to the aggregate resource constraint

\[
c(y^t) + b(y^t) = \sum_{i=1}^{2} y_{it} + (1 + r) b(y^{t-1}),
\]

for all \(y^t, t \geq 0\), to the enforcement constraint

\[
\sum_{\tau=t}^{\infty} \sum_{y^\tau} \beta^{t-\tau} \pi(y^\tau | y^t) u(c(y^\tau)) \geq V_{aut}(y^t)
\]

for all \(y^t, t \geq 0\), where

\[
V_{aut}(y^t) = \sum_{\tau=t}^{\infty} \sum_{y^\tau | y^t} \beta^{t-\tau} \pi(y^\tau | y^t) u \left( \left(1 - \phi \right) \sum_{i} y_{ir} \right),
\]

for all \(y^t, t \geq 0\), and to the no-Ponzi game condition (4.1).

Given the optimal plan \(c(y^t)\) from step 1, step 2 solves for the optimal distribution of aggregate consumption among the union partners. Formally, the step 2 problem is

\[
\max_{c_i(y^t)} \sum_{i} \lambda_i \sum_{t=0}^{\infty} \sum_{y^t} \beta^t \pi(y^t) u(c_i(y^t))
\]

subject to

\[
\sum_{i} c_i(y^t) = c(y^t),
\]

for all \(y^t, t \geq 0\).

With isoelastic preferences, the step 2 problem admits a simple, explicit solution. It is relatively easy to show that

\[
c_i(y^t) = \alpha_i c(y^t)
\]

where \(\alpha_i \equiv \lambda^{1/\sigma}_i / \sum_j \lambda^{1/\sigma}_j\), for \(i = 1, 2\). That is, individual consumption is a constant fraction of aggregate consumption. The fraction is increasing in the country’s welfare weight.

Similarly to Section 3.2, the step 1 planner’s problem admits a recursive formulation:

\[
V^U(\bar{b}, \bar{y}) = \max_{c, \bar{b}'} \left\{ u(c) + \beta \sum_{y'} \pi(y' | y) V^U(\bar{b}', \bar{y}') \right\}
\]

(P1')
subject to

\[ c + \bar{b}' = \sum_i y_i' + (1 + r)\bar{b} \]

\[ \bar{b}' \geq \bar{b}^U(\bar{y}) \]

where

\[ \bar{b}^U(\bar{y}) = \max_{\bar{y}' : \#(\bar{y}'|\bar{y}) > 0} \{ b'_{\bar{y}'} : V^U(\bar{b}, \bar{y}') = V^U_{aut}(\bar{y}') \} \]

(4.5)

and where \( V^U_{aut}(\bar{y}) \) solves

\[ V^U_{aut}(\bar{y}) = u \left( 1 - \phi \right) \sum_i y_i + \beta \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y})V^U_{aut}(\bar{y}'). \]

Given (4.4), the value for country \( i \) of belonging to a union with country \( j \) is

\[ V^U_i(\bar{b}, \bar{y}) = \alpha_i^{1-\sigma} V^U(\bar{b}, \bar{y}). \]

(4.6)

4.2 Competitive equilibrium

To perform our welfare analysis, we still need to recover the planner’s welfare weights as a function of the initial pair of union partner states.

We use Negishi’s (1960) iterative method to compute these welfare weights. This well-known method exploits the first welfare theorem, which allows us to obtain the competitive equilibrium allocation as the solution to the planner’s problem for a given set of welfare weights. By requiring that the planner’s allocation be affordable under the equilibrium prices, we obtain the unique pair of welfare weights that lead to the competitive equilibrium allocation associated with a given set of initial states.

We need to consider a decentralization of the constrained efficient allocation. We consider a competitive equilibrium with tax subsidies, in line with Wright (2006). The decentralization works as follows. Within the union, countries trade a complete set of Arrow securities. In world credit markets, they trade freely on non-contingent bonds. However, a central government authority in the union taxes each country’s income in a lump-sum fashion, and uses the proceeds to subsidize asset purchases. The government’s tax and transfer policy is designed to support the constrained-efficient allocation. A subsidy is required to encourage union partners to save in those states when they would be inclined to default. Our procedure is described in more detail in Appendix B.
4.3 Discussion

Several features of union formation in our model are worth discussing. The role of initial conditions when computing the welfare gains from financial market integration is a crucial feature of our analysis. Whether a country is rich or poor at the time of union formation is a key determinant of the sign of the welfare gains. In the international risk-sharing literature, the role of initial conditions has sometimes been sidestepped (Cole and Obstfeld, 1991; van Wincoop, 1999; Athanasoulis and van Wincoop, 2000, either impose symmetry, or look at a representative country), whereas in other papers (van Wincoop, 1994; Lewis, 2000; Imbs and Mauro, 2008) it is allowed to play a role. Differently from this literature, however, in our model union formation may entail a welfare loss. This generates the potential for disagreement about union formation. We exploit this by requiring that unions be formed only when both partners experience a welfare gain, given the initial conditions set in the world economy. That is, union formation in our model requires unanimity.

For a large set of country pairs in our model, unions only lead to potential Pareto improvements, with one country losing. This raises the possibility of introducing side payments to compensate the losers. Our analysis abstracts from such transfer schemes. In our setup, wealth would need to be redistributed away from poor and toward rich partners. We suspect the implementation of such schemes would face strong opposition in poor countries. Moreover, we do not have evidence from actual integration arrangements suggesting such schemes have taken place. Finally, we believe it is more appropriate to focus our analysis strictly on the benefits from risk-sharing, separately from side-payments.

Rather than implementing a pure transfer scheme, the two partners could instead agree ex-ante on distorting the baseline union allocation, tilting it to the benefit of rich partners. Formally, one would impose participation constraints at the time of union formation, such that every partner may potentially benefit from it. This would increase the likelihood of union formation among heterogeneous partners, at the expense of future risk-sharing benefits. Presumably, such arrangement would be easier to implement compared to a pure transfer scheme. We think it would be very interesting to extend our analysis along this dimension. We still prefer to focus on the strict benefits from risk-sharing in this paper, and consider the role of initial participation constraints and their implementation in future research.\(^6\)\(^7\)

---

\(^6\)In the European Union, the Cohesion Fund is a transfer scheme that takes the exact opposite form: resources are transferred from rich to poor members.

\(^7\)From a technical standpoint, we would have to develop a different decentralization of the constrained-efficient
We considered unions with *centralized international trade and default decisions*. An alternative setting is one in which each individual member country unilaterally decides whether to default. Jeske (2006) provides an analysis of this situation. As Section 4.1 makes clear, a major advantage of our centralized setting is analytical convenience, since it does not require solving directly for the market allocation. Note however that with decentralized default, potentially defaulting union members presume continued indirect access to world markets, by using the remaining non-defaulting members as intermediaries. This increases the incentives to default, and therefore tightens borrowing limits within the union relative to centralized default. All else constant, union formation is thus even less likely under decentralized compared to centralized default. Our analysis can be thought of as giving the best chance for union formation.

For tractability, our analysis restricts attention to *two-country unions*. In our model, since endowment risk is purely idiosyncratic, additional partners would be potentially beneficial to the union since they would further enhance risk-sharing opportunities. However, solving the frictions among union members is also likely to become more difficult and costly as the number of partners increases. This is precisely the starting premise of our paper, that solving frictions is easier at a smaller scale. Our model could be extended by introducing a cost of union formation that is increasing with the number of countries.\(^8\) Such a setting would deliver implications for both the number and the type of countries most likely to form a union. We leave the analysis of these interesting implications to future research.

Finally, a country pair contemplating union formation is given a take-it-or-leave choice at time 0. If the union is formed, it is assumed to be forever enforced. Our analysis abstracts away from the important issue of sustainability of the economic union. Although *union breakups* are very rare in the data, they can be ex-post optimal in our model, depending on the endowment realization. Without an enforcement technology, sustaining the union would require distorting the optimal allocation, to ensure that the relevant ex-post participation constraints are met. In some cases this might not be possible, leading to a breakup of the union. See Fuchs and Lippi (2006) for an analysis of the sustainability of monetary unions with some of these features.

---

\(^{8}\)Imbs and Mauro (2008) find that, regarding benefit side alone, most risk-sharing gains would be achievable in unions of seven member-countries or less. Further, in our model it is difficult for a large number of countries to all agree about union formation. This suggests that even very small costs would be sufficient to generate to small-scale arrangements.
5 Results

Our goal is to characterize which country pairs find it individually rational to form a union. The main benefit of union formation is the possibility of sharing risk with a partner. There are also costs, however. First, default becomes more attractive for union members, since they may still share risk upon default. As a result, borrowing constraints are tighter in the union. In our benchmark calibration, the borrowing limit increases from $b_W^i = -0.302$ in the world economy, to $\bar{b}_i^U = \bar{b}^U/2 = -0.235$ in the union, on a per country basis.

Second, in asymmetric unions, poorer country members tend to borrow heavily from the rest of the world, and exhaust the whole union’s borrowing limit. This creates a negative externality for richer countries, which find themselves more frequently borrowing-constrained compared to standing alone in the world economy. Although being part of an asymmetric union tends to be beneficial for poorer members, it also tends to generate losses for richer countries. Our model will therefore produce a bias against forming asymmetric unions.

We now turn to a more detailed analysis of union formation. We compute the welfare gain for each country of forming a union with a specific partner in terms of consumption equivalents. That is, as the percentage increase in consumption, constant across time and states of nature, that leaves the country indifferent between standing alone in the world economy and forming the union.

Consider two countries sitting in the world economy at time 0, with states $(b_{i0}, y_{i0})$, $i = 1, 2$. If they form a union, the initial aggregate state is $(\bar{b}_0, \bar{y}_0)$, with $\bar{b}_0 = b_{10} + b_{20}$ and $\bar{y}_0 = (y_{10}, y_{20})$. Let $c_W^{i} (b_{i0}, y_{i0})$ represent a state-contingent consumption stream for country $i$ in the world economy, from state $(b_{i0}, y_{i0})$ onwards. Let $c_U^i (\bar{b}_0, \bar{y}_0)$ represent a state-contingent consumption stream for country $i$ if both countries decide to form a union at time 0. Let $U(c_W^{i} (b_{i0}, y_{i0}))$ and $U(c_U^i (\bar{b}_0, \bar{y}_0))$ denote the expected lifetime utility derived from these consumption streams. Now denote by $(1 + \mu_{ij})c_W^{i} (b_{i0}, y_{i0})$ the consumption stream derived from $c_W^{i} (b_{i0}, y_{i0})$, where every state-contingent consumption level is increased by $\mu_i$ percent. The welfare gain for country $i$ of forming a union with country $j$ is the $\mu_{ij}$ that solves:

$$U((1 + \mu_{ij})c_W^{i} (b_{i0}, y_{i0})) = U(c_U^i (\bar{b}_0, \bar{y}_0)),$$
or, with isoelastic preferences as in (3.7),

\[
\mu_{ij} = \left[ \frac{U(c^U_i(\bar{b}_0, \bar{y}_0))}{U(c^W(b_i, y_i))} \right]^{\frac{1}{1-\sigma}} - 1
\]

\[
= \left[ \frac{V_i^U(\bar{b}_0, \bar{y}_0)}{V(b_i, y_i)} \right]^{\frac{1}{1-\sigma}} - 1,
\tag{5.1}
\]

where the value functions have been defined in (P0) and (4.6). Notice that our welfare numbers incorporate transitional dynamics.

We next study the separate roles of wealth heterogeneity and wealth levels for union formation.

### 5.1 Role of wealth heterogeneity

Figure 1 displays the welfare gain for country 1 of forming a union, as a function of country 1 and country 2’s initial net foreign asset levels. The figure is conditional on both countries starting the union formation process with mid-level endowment, \(y_m\). Union partners are heterogenous only in terms of initial debt levels.

Several observations emerge from Figure 1. First, country 1 experiences a welfare loss for a large range of net foreign asset levels. The equilibrium welfare gains range from -1.4\% to 3.7\%, with a mean of 0.5\%. These are low welfare gains from union formation. Comparing with the literature on the welfare gains from international risk-sharing, the average gain is similar to the values in the lower end of the range, as summarized by van Wincoop (1999), and in line with those obtained by Cole and Obstfeld (1991), Backus, Kehoe, and Kydland (1992), Obstfeld (1994a), Tesar (1995), and Mendoza (1995).

Second, Figure 1 shows that country 1’s welfare gain is always increasing in the partner’s net foreign assets. Third, country 1’s welfare gain is increasing in own net foreign assets only if the partner’s is sufficiently low;\(^9\) otherwise, if the partner is rich, the welfare gain is monotonically decreasing in own net foreign assets. Put together, the last two observations suggest the key determinant for union formation is the amount of the resources the partner has: a country would like to belong to a rich club, especially if it’s poor.

Figure 2 displays the agreement areas, i.e. the set of initial country states for which both countries would experience a welfare gain, and thus agree to form a union. Figure 2 is restricted to endowment levels in \(\{y_l, y_m, y_h\}\). For states above the solid lines, country 1 would improve welfare

\(^9\)Although not apparent from the Figure 1, the welfare gain is actually non-monotonic in own net foreign assets if the partner’s is low enough.
by forming a union with country 2, and similarly for country 2 for states below the dashed lines. The agreement areas are therefore represented by the light-shaded areas.

Superimposed on Figure 2 is also an area representing the ergodic space for net foreign asset positions in the world economy, $b_{10}, b_{20} \in [-0.302, 3.869]$. This is the dashed square located inside each figure. Notice the role played by the world equilibrium in our analysis of union formation. It determines both the world interest rate faced by the union, and also the relevant subset of country pairs that are faced with the option union formation.

We begin with the first row of Figure 2. In this row, potential union members have identical initial endowments, but potentially different wealth levels. The figure shows, first, that unions tend to be formed between countries sufficiently homogeneous in terms of initial wealth. Along the 45 degree line, and restricted to the ergodic space, countries always reach an agreement. The disagreement area exists when wealth levels are sufficiently different from each other. Second, we also see that whenever partners disagree, the rich are the ones with a potential welfare loss. They

---

10Since the average endowment is equal to 1, these quantities correspond also to net foreign assets to average output ratios.
are the ones preventing union formation.

Turning now to the bottom row of Figure 2, which corresponds to asymmetric initial endowments, we see that endowment heterogeneity makes it nearly impossible for countries to agree to form a union. Indeed, restricting to heterogeneous endowment levels in \( \{y_l, y_m, y_h\} \), an agreement is never reached. Although country 1, the endowment-poor country, would always benefit from union formation (the ergodic space is always above the solid line), this is not the case for country 2, the endowment-rich country. Only a sufficiently asset-poor country 2 would like to form a union with an endowment-poor country 1. This effect is less dramatic the less asymmetric the initial endowment levels are. For example, some agreements may be supported with \( (y_{10}, y_{20}) = (y_{mh}, y_h) \), depending on the initial net foreign asset levels.

The bottom line is that country homogeneity, either in terms of net foreign assets or endowments, is a key determinant of union formation. Unions are more likely to form among similar countries.
5.2 Role of wealth levels

We now turn to the role of wealth (net foreign assets plus endowment) levels. From the first row of Figure 2, we see that a larger union-wide endowment favors union formation. First because, as we move from the left to the right panel, the agreement area fills a larger area of the ergodic space. Second because the agreement areas get wider for larger wealth levels, which is particularly noticeable when conditional on \((y_l, y_l)\).

We summarize the discussion of this and the previous subsection with the following. Unions are more likely to be formed:

1. the wealthier the partners, and
2. the more homogeneous the partners,

either in terms of initial endowment or net foreign assets. Quantitatively, the most important determinant of union formation appears to be partner homogeneity.

5.3 Quantitative implications

To explore the quantitative implications of the model, we compute the probability of union formation conditional on different regions of the state-space.\(^{11}\) We ask: What is the probability that two randomly-picked countries from particular subsets of the world distribution agree to form a union?

In selecting subsets of the ergodic space, we focus on the top and bottom terciles for output (respectively defined as \(Y_h = [y_{2/3}, y_{\text{max}}]\) and \(Y_l = [y_{\text{min}}, y_{1/3}]\)) and net foreign-assets over GDP (respectively defined as \(B_h = [(b/y)_{2/3}, (b/y)_{\text{max}}]\) and \(B_l = [(b/y)_{\text{min}}, (b/y)_{1/3}]\)). We define such sets in the exact same way both in the actual data and in the model. Since the results are similar across our empirical definitions of unions, in the actual data we restrict attention to customs unions or deeper arrangements.

We restrict attention to only three subsets, with the aim of capturing the key implications we drew from Figure 2. More specifically, take country pairs defined by their current output and net foreign assets over GDP.\(^{12}\) We consider “Rich” country pairs (both in the set \(Y_h \times B_h\)), “Poor”

\(^{11}\)An alternative procedure would be to run a probit-gravity regression on artificial data which would be the exact analogue of the one in Section 2, except that the terms involving geography and scale would be excluded. Unfortunately, due the nonlinear nature of the regression model, the marginal effects would be hard to compare. They would be a function not only of the estimated parameters, but also of the data itself (actual vs artificial).

\(^{12}\)For the reasons explained in Section 2, by “current” levels we actually mean five-year averages.
country pairs (both in the set \( Y_l \times B_l \)), and “Unequal” country pairs (one in the set \( Y_h \times B_h \) and the other in \( Y_l \times B_l \)). We also compute the “Unconditional” probability of union formation.

<table>
<thead>
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<th>Data</th>
<th>Data, common border</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich</td>
<td>16%</td>
<td>71%</td>
<td>68%</td>
</tr>
<tr>
<td>Poor</td>
<td>8%</td>
<td>20%</td>
<td>43%</td>
</tr>
<tr>
<td>Unequal</td>
<td>0%</td>
<td>0%</td>
<td>5%</td>
</tr>
<tr>
<td>Unconditional</td>
<td>4%</td>
<td>32%</td>
<td>40%</td>
</tr>
</tbody>
</table>

Table 3: Conditional Probabilities of Union Formation

Our results are summarized in Table 3. The first column pertains to the entire data set. Only 4% of all country pairs are part of a customs union or deeper arrangements. This number is 8% conditional on poor country pairs, and 16% conditional on rich country pairs. The data does not feature unions among unequal pairs.

The second column repeats these calculations restricting to country pairs sharing a common border. The results are qualitatively similar, but the conditional probabilities of union formation are now much higher. The conditional probabilities in the model are in the third column. They are reasonably close to the empirical probabilities conditional on countries sharing a common border. Since our model abstracts from geography as a determinant of union formation, we find it more appropriate to compare the model’s implications to the data restricted to common border countries. The main discrepancy is that our model implies low wealth levels are not as detrimental to union formation compared to the data. Poor countries in the model are twice as likely to form a union compared with the data.

We conclude that our model seems to provide a reasonably accurate description of the incentives for union formation, namely the role of wealth levels and wealth inequality.

6 Conclusion

We have developed a quantitative theory of economic integration based on the incentives to share income risk. We have modeled an economic union as a small-scale arrangement that solves the frictions that otherwise limit the extent of risk sharing in the world economy.
Our model emphasizes not only the risk-sharing benefits of union formation, but also its costs. One cost is that, a country that is part of a union will not be able to borrow as much as a country standing alone in the world economy. This is because unions have larger incentives to default. Another cost is for rich countries forming a union with poor countries. Poor countries tend to exhaust the whole union’s credit limit, imposing a negative externality on rich countries. Our model implies that economic integration should not happen very often, and when unions do get formed it is mostly among rich and homogeneous countries. These features appear to be consistent with real-world arrangements.

Our paper has abstracted from a host of issues that could be potentially important for union formation based on risk-sharing. In particular, we have assumed different countries are characterized by common and independent income processes. In reality, shocks tend to be correlated among subsets of countries, which would work against union formation in our model.\textsuperscript{13} Further, there is large cross-country heterogeneity in income risk, with poorer countries being more volatile (\textsc{Acemoglu and Zilibotti} (1997)). In our model, this could potentially increase the likelihood of union formation among poor countries.\textsuperscript{14} Finally, there are also differences in country size. All these issues deserve further scrutiny.

Our paper has also focused on just one particular dimension of economic integration, the sharing of risk. It would be interesting to consider other key dimensions of economic integration within small scale arrangements, namely liberalizing goods flows (\textsc{Melitz} (2003), \textsc{Alvarez and Lucas} (2007)), labor flows (\textsc{Klein and Ventura} (2007)), and investment flows (\textsc{Castro} (2005), \textsc{Gourinchas and Jeanne} (2008), \textsc{Burstein and Monge-Naranjo} (2009), \textsc{McGrattan and Prescott} (2009)).\textsuperscript{15}

\textsuperscript{13}Instead, correlated shocks is traditionally emphasized as a motivation for the formation of currency unions.

\textsuperscript{14}A similar implication follows from \textsc{Imbs and Mauro}'s (2008) analysis.

\textsuperscript{15}Further dimensions of small scale economic integration that received some attention in the recent literature include adopting a common currency (\textsc{Alesina and Barro} (2002)) and coordinating public policy (\textsc{Alesina, Angeloni, and Etro} (2005)).
A Data

A.1 Regional Agreements

The list of regional trade agreements that we use in the regression analysis of Section 2, by type, and their country composition, is as follows:

- **Economic Unions.** *Economic and Monetary Community of Central Africa* (Cameroon, Central African Republic, Chad, Congo D.R., and Equatorial Guinea), *Euro zone* (Austria, Belgium, Luxembourg, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain), and *West African Economic and Monetary Union* (Benin, Burkina Faso, Guinea-Bissau, Côte d’Ivoire, Mali, Niger, Senegal, and Togo).

- **Common Markets.** In addition to all Economic unions: *East African Community* (Kenya, Tanzania, and Uganda), and *European Economic Area* (comprising the European Free Trade Area of Iceland, Liechtenstein, and Norway, plus all the countries in the EU25).

- **Customs Unions.** In addition to all Common Markets: *Andean Community* (Bolivia, Colombia, Ecuador, Peru, and Venezuela), *Caribbean Community* (Antigua and Barbuda, Bahamas, Barbados, Belize, Dominica, Grenada, Guyana, Jamaica, Montserrat, Saint Kitts and Nevis, Saint Lucia, Saint Vincent and the Grenadines, Suriname, and Trinidad and Tobago), *Eurasian Economic Community* (Belarus, Kazakhstan Kyrgyzstan, Russia, and Tajikistan), *EU25-Turkey* (all the countries in the EU25 plus Turkey), *Gulf Cooperation Council* (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and United Arab Emirates), *Southern Common Market* (Argentina, Brazil, Paraguay, and Uruguay), and *South African Customs Union* (Botswana, Lesotho, Namibia, South Africa, and Swaziland).

A.2 Countries

The full sample of 136 countries that we use in the regression analysis of Section 2 includes: Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bahrain, Bangladesh, Belgium, Belize, Benin, Bhutan, Bolivia, Brazil, Bulgaria, Burkina Faso, Burundi, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Comoros, Congo D.R., Congo Rep., Costa Rica, Côte d’Ivoire, Cyprus, Czech Republic, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, Gabon, Gambia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras,
Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Kiribati, Korea, Kuwait, Lao PDR, Liberia, Madagascar, Malawi, Malaysia, Mali, Malta, Mauritania, Mauritius, Mexico, Mongolia, Morocco, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Rwanda, Samoa, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, Solomon Islands, South Africa, Spain, Sri Lanka, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Sudan, Sweden, Switzerland, Syrian Arab Republic, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Venezuela, Vietnam, Yemen, Zambia, and Zimbabwe.

B Decentralization

We decentralize the planner’s allocation as a competitive equilibrium with tax subsidies on saving. Our decentralization scheme is an adaptation of Wright (2006).\(^\text{16}\) Within the union, countries trade a complete set of Arrow securities. In the world market, they trade freely on a riskless one-period bond. A central government authority in the union implements a tax and transfer scheme, designed to support the constrained-efficient allocation, and thus prevent default in the appropriate states.

For each country \(i = 1, 2\) in the union, let \(a_i(y'; b_i, \bar{b}, \bar{y})\) denote the net stock of the Arrow security that pays in state \(\bar{y}'\) tomorrow, conditional on individual wealth \(b_i\) and the aggregate state \((\bar{b}, \bar{y})\), with price \(q(y'; \bar{b}, \bar{y})\). Let \(b_i'(b_i, \bar{b}, \bar{y})\) denote the net stock of foreign bonds that earn interest \(r\) tomorrow.

Let also \(\tau(\bar{b}, \bar{y})\) denote the subsidy rate on net asset purchases, and \(T_i(b_i, \bar{b}, \bar{y})\) the lump-sum income tax faced by country \(i\).

In a competitive equilibrium with capital controls, country \(i\) solves the following problem for every current state

\[
V_i(b_i, \bar{b}, \bar{y}) = \max_{c_i, b_i', a_i(y')} \left\{ u(c_i) + \beta \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) V_i(b_i', \bar{b}, \bar{y}') \right\}
\]

\(^{16}\)Wright (2006) uses taxes on borrowing instead of saving subsidies, although the two are equivalent. Wright (2006) also studies an alternative decentralization based upon country-specific borrowing limits, along the lines of Alvarez and Jermann (2000).
subject to

\[ c_i + (1 - \tau(\bar{b}, \bar{y})) \left( b'_i + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}') \right) = b_i + T_i(b_i, \bar{b}, \bar{y}) \quad (B.1) \]

and to a perceived law of motion for aggregate foreign asset holding \( \bar{b} \).

The government is assumed to run a balanced budget for each country separately, that is

\[ \tau(\bar{b}, \bar{y}) \left( b'_i (b_i, \bar{b}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}'; b_i, \bar{b}, \bar{y}) \right) = T_i(b_i, \bar{b}, \bar{y}) \quad (B.2) \]

for every current state and for each \( i \).

A competitive equilibrium with tax subsidies is defined in the standard way, as (i) optimal decision rules that solve each country’s problem given prices, government policy, and a perceived law of motion for aggregate wealth; (ii) a government policy that satisfies the balanced budget constraints given prices and individual decisions; (iii) Arrow security prices that clear asset markets; and (iv) consistency between the perceived law of motion for aggregate asset holding and the individual decision rules.

Our goal here is to show that there exists a government tax and transfer policy that supports the constrained-efficient allocation as a competitive equilibrium. We focus on the key steps of the argument.

Consider the first-order conditions to the country’s problem

\[ 1 - \tau(\bar{b}, \bar{y}) = (1 + r) \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) \beta u' \left( c_i(b'_i, \bar{b}', \bar{y}') \right) \frac{a_i(b'_i, \bar{b}', \bar{y}')}{u'(c_i(b_i, \bar{b}, \bar{y}))} \quad (B.3) \]

\[ (1 - \tau(\bar{b}, \bar{y})) q(\bar{y}'; \bar{b}, \bar{y}) = \pi(\bar{y}'|\bar{y}) \beta u' \left( c_i(b'_i, \bar{b}', \bar{y}') \right) \frac{a_i(b'_i, \bar{b}', \bar{y}')}{u'(c_i(b_i, \bar{b}, \bar{y}))}. \quad (B.4) \]

Given isoelastic preferences, the last equation implies

\[ \frac{c_i(b'_i, \bar{b}', \bar{y}')}{c_i(b_i, \bar{b}, \bar{y})} = \frac{c(b'_i, \bar{y}')}{{c}(b, \bar{y})} \quad \text{for } i = 1, 2. \quad (B.5) \]

The two Euler equations imply

\[ 1 = (1 + r) \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}). \quad (B.6) \]

Note also that, at the optimum, we may use (B.2) to eliminate subsidies and transfers from (B.1):

\[ c_i(b_i, \bar{b}, \bar{y}) + b'_i(b_i, \bar{b}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}'; b_i, \bar{b}, \bar{y}) = b_i. \quad (B.7) \]
Consider now the constrained-efficient allocation, the solution to problem (P1'). This allocation, which we denote with a star superscript, satisfies the planner’s Euler equation
\[ u'(c^*(\bar{b}, \bar{y})) - \phi^*(\bar{b}, \bar{y}) = \beta(1 + r) \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) u'(c^*(\bar{b}', \bar{y}')). \]  
(B.8)

Using (B.5) in (B.3), and requiring that the resulting allocation be consistent with (B.8), it is easy to compute the state-contingent subsidy rates that implement the constrained-optimal allocation as
\[ \tau(\bar{b}, \bar{y}) = \phi^*(\bar{b}, \bar{y}) \frac{u'(c^*(\bar{b}, \bar{y}))}{u'(c^*(\bar{b}, \bar{y}))}. \]  
(B.9)

Note that if the borrowing constraint to problem (P1') does not bind in state (\(\bar{b}, \bar{y}\)), then \(\phi^*(\bar{b}, \bar{y}) = 0\) and so \(\tau(\bar{b}, \bar{y}) = 0\). In this case, from (B.4) and (B.6), the domestic interest rate equals the world interest rate. If the constraint is instead binding, then the (post-subsidy) domestic interest rate is higher than the world interest rate. This ensures that countries save in a constrained-optimal way, and that equilibrium borrowing is self-enforcing.

It is relatively straightforward to show formally that, given a constrained-efficient allocation that solves (P1') and (P2) for the appropriate set of welfare weights, one can obtain individual asset holdings from (B.7) together with the market clearing condition for Arrow securities, Arrow security prices from (B.4), and a government policy from (B.9) and (B.2) that support that allocation as a competitive equilibrium with tax subsidies.

To find the appropriate set of welfare weights, we use the method proposed by Negishi (1960). This method exploits the equivalence between the market and the constrained-efficient allocations.

We obtain the time-0 present value budget constraint of country \(i\) by iterating forward on the flow budget constraint (B.7). We express it as
\[ C_i(b_{i0}, \bar{b}_0, \bar{y}_0) = Y_i(\bar{b}_0, \bar{y}_0) + (1 + r) b_{i0}, \]
where \(C_i(b_{i0}, \bar{b}_0, \bar{y}_0)\) and \(Y_i(\bar{b}_0, \bar{y}_0)\) are the time-0 present-values of consumption and the endowment, respectively. At time 0, the time of forming the union, \(\bar{y}_0\) is the union’s endowment pair, \(b_{i0}\) is country \(i\)’s net stock of foreign bonds, and \(\bar{b}_0 = \sum_i b_{i0} = \sum_i y_{i0} + (1 + r) \sum_i b_{i0}\) is the union’s aggregate wealth.

It follows from (4.4) that we may express the present value of individual consumption as fraction of the present value of aggregate (constrained-efficient) consumption, that is \(C_i(b_i, \bar{b}, \bar{y}) = \alpha_i C^*(\bar{b}, \bar{y})\). Replacing above allows us to recover the individual consumption share parameters as
\[ \alpha_i = \frac{(1 + r) b_{i0} + Y_i(\bar{b}_0, \bar{y}_0)}{C^*(\bar{b}, \bar{y})}. \]  
(B.10)
Given equilibrium Arrow security prices \( q (\bar{y}'; \bar{b}, \bar{y}) \), and optimal decision rules \( c^* (\bar{b}, \bar{y}) \) and \( b^* (\bar{b}, \bar{y}) \), the \( C^* \) and \( Y \) functions solve the following functional equations

\[
Y_i (\bar{b}, \bar{y}) = y_i + \sum_{\bar{y}'} q (\bar{y}'; \bar{b}, \bar{y}) Y_i (\bar{b}', \bar{y}') \quad \text{(B.11)}
\]

\[
C^* (\bar{b}, \bar{y}) = c^* (\bar{b}, \bar{y}) + \sum_{\bar{y}'} q (\bar{y}'; \bar{b}, \bar{y}) C^* (\bar{b}', \bar{y}') \quad \text{(B.12)}
\]

with

\[
\bar{b}' = \sum_i y_i' + (1 + r) b^* (\bar{b}, \bar{y}) .
\]

Notice that although it is straightforward to obtain the welfare weights from the consumption share parameters, we only need to know the \( \alpha_i \)'s in order to uncover the individual allocations.

**C Numerical algorithms**

**C.1 World economy equilibrium**

Our algorithm can be described in the following steps:

1. Solve for the autarky value function \( V_{aut}(y) \) from equation (3.6).

2. Given a current guess for the equilibrium interest rate \( r \), solve problem (P0) by iterating on the following steps:

   (a) Consider the \( n \)th iteration, with a current conjecture for the debt limit \( b_n^W \). For the initial conjecture, we use the natural borrowing constraint.

   (b) Given \( b_n^W \), solve problem (P0) by policy function iteration. We discretize the state-space and use cubic-spline interpolation to compute decisions outside the grid.

   i. First find the decision rules that solve the system of first-order conditions to problem (P0), ignoring the debt limit. Consider the \( j \)th iteration, with a current conjecture for the consumption decision rule \( c_j^i (b, y) \). Compute a candidate update \( c_{j+1}^i (b, y) \) by solving

   \[
u' (c_{j+1}^i (b, y)) = \beta (1 + r) \sum_{y'} \pi(y'|y) u' (c_j^i (b', y'))\]

   with

   \[b' = y + (1 + r) b - c_{j+1}^i (b, y).\]

   As part of the solution, we obtain \( b_{j+1}^W (b, y) \).
ii. Check whether the borrowing constraint is violated. If \( b_{n+1}^{ij+1}(b, y) < b_n^W \), then update the solution as follows:

\[
\begin{align*}
    b_{n+1}^{ij+1}(b, y) &= b_n^W \\
    c_{n+1}^{ij+1}(b, y) &= b - b_{n+1}^{ij+1}(b, y) \\
    \phi_{n+1}^{ij+1}(b, y) &= u'(c_{n+1}^{ij+1}(b, y)) - \beta(1 + r) \sum_{y'} \pi(y'|y) u'(c_{n+1}^{ij+1}(b', y')),
\end{align*}
\]

If instead \( b_{n+1}^{ij+1}(b, y) \geq b_n^W \), then update using the unconstrained solution, setting also \( \phi_{n+1}^{ij+1}(b, y) = 0 \).

iii. Iterate on the previous two steps until the decision rules converge. At the end, compute the value function \( V_n(b, y) \).

(c) Given \( V_n(b, y) \), update the debt limit as follows:

\[
    b_{n+1}^W = \max_{y'} \{ b_{y'} : V_{n}(b_{y'}, y') = V_{aut}(y') \}.
\]

(d) Iterate on steps 2b and 2c until the borrowing limits converge.

3. Check the market clearing condition by approximating the aggregate bond holding in the world economy with the total bond holding of a particular country over a very long simulation period. We discretize the state-space using a finer grid, and linearly interpolate the decision rules.

4. Iterate on steps 2 and 3 until we find an interest rate that approximately clears the bond market.

C.2 Union problem under centralized default

Our algorithm to solve for the union’s allocation given an equilibrium world interest rate \( r \) can be described as follows:

1. Solve problem (P1′) using the method described in step 2 of the algorithm of Section C.1. As part of the solution we obtain the union decision rule \( c^*(\bar{b}, \bar{y}) \), the multiplier function \( \phi^*(\bar{b}, \bar{y}) \), and the value function \( V^U(\bar{b}, \bar{y}) \).
2. Decentralize the union’s constrained-efficient allocation as a competitive equilibrium with capital controls.
(a) Compute tax-subsidies from (B.9).

(b) Compute pre-subsidy Arrow-security prices from (B.4).

(c) Compute the present-value functions from (B.11) and (B.12). In practice, we guess some arbitrary functions on a grid and then iterate on the two recursive equations until convergence. We linearly interpolate these functions when future wealth levels fall outside the grid.

(d) Compute consumption shares from (B.10).

(e) Compute the value function for each country from (4.6).
References


(2009): “Database of Economic Integration Agreements,” University of Notre Dame.


